M1B/Schoenbrun Fundamental Theorm

Let $F(u) = \int_a^u f(t)\,dt$

so by the fundamental theorem of calculus

$$\frac{dF}{du} = f(u)$$

Now let $u = g(x)$

so

$$\frac{du}{dx} = g'(x)$$

By the chain rule

$$\frac{dF}{dx} = \frac{dF}{du} \cdot \frac{du}{dx} = f(u) \cdot \frac{du}{dx} = f\left(g(x)\right) g'(x)$$

so we have the result

$$\frac{d}{dx} \int_a^{g(x)} f(t)\,dt = f\left(g(x)\right) \cdot g'(x)$$

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Example:

$$F(u) = \int_a^u \sin(t)\,dt$$

if $u = x^2$

$$\frac{d}{dx} \int_a^{x^2} \sin(t)\,dt = \sin\left(x^2\right) \cdot 2x$$