Section 7.1 Page 514 (15, 17, 23, 30, 35, 38)

15)

Multiply by $\frac{e^x}{e^x}$

$$\int \frac{e^x}{e^x - 2e^{-x}}\,dx = \int \frac{e^{2x}}{e^{2x}-2}\,dx$$

Substitute $u = e^{2x} \to du = 2e^{2x}dx$

$$\int \frac{e^{2x}}{e^{2x}-2}\,dx = \frac{1}{2}\int \frac{du}{u-2} = \frac{1}{2}\ln|u-2| + C = \frac{1}{2}\ln\left|e^{2x}-2\right| + C$$

17)

$$\int_1^{e^2} \frac{\ln^2\left(x^2\right)}{x}\,dx = \int_1^{e^2} \frac{\ln\left(x^2\right)\ln\left(x^2\right)}{x}\,dx = \int_1^{e^2} \frac{2\ln(x)2\ln(x)}{x}\,dx = 4\int_1^{e^2} \frac{\ln^2(x)}{x}\,dx$$

Substitute $u = \ln x \to du = \frac{1}{x}dx$

$$4\int_1^{e^2} \frac{\ln^2(x)}{x}\,dx = 4\int_1^{e^2} \frac{\ln^2(x)}{x}\,dx = 4\int_0^{\ln e^2} u^2\,du = 4\left[\frac{u^3}{3}\right]_0^2 = 4\left[\frac{8}{3} - 0\right] = 32/3$$

23)

$$\int \frac{x+2}{x^2+4}\,dx = \int \frac{x}{x^2+4}\,dx + \int \frac{2}{x^2+4}\,dx$$

For the first integral substitute $u = x^2 + 4 \to du = 2x\,dx$

$$\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + C = \frac{1}{2}\ln\left(x^2+4\right) + C$$

For the second integral, substitute $x = 2u \to dx = 2du$

$$\int \frac{2}{x^2+4}\,dx = \int \frac{4}{4u^2+4}\,du = \arctan u + C = \arctan\frac{x}{2} + C$$

30)

$$\int_2^4 \frac{x^2+2}{x-1}\,dx = \int_2^4 x + 1 + \frac{3}{x-1}\,dx = \left[\frac{x^2}{2} + x + 3\ln|x-1|\right]_2^4 =$$

$$\left[8 + 4 + 3\ln 3 - (2 + 2 + 3\ln 1)\right] = 8 + 3\ln 3$$

35)

$$\int \frac{d\theta}{\sqrt{27-6\theta-\theta^2}} = \int \frac{d\theta}{\sqrt{36-\left(\theta^2+6\theta+9\right)}} = \int \frac{d\theta}{\sqrt{36-\left(\theta+3\right)^2}}$$

Substitute $u = \frac{\theta+3}{6} \rightarrow du = \frac{1}{6}d\theta$

$$\int \frac{d\theta}{\sqrt{36-\left(\theta+3\right)^2}} = \int \frac{6du}{\sqrt{36-36u^2}} = \int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C = \arcsin\left(\frac{\theta+3}{6}\right) + C$$

38)

$\int \frac{1-x}{1-\sqrt{x}}dx$ Multiply by $\frac{1-\sqrt{x}}{1-\sqrt{x}}$

$$\int \frac{1-x}{1-\sqrt{x}}dx = \int \frac{1-x}{1-\sqrt{x}} \cdot \frac{1-\sqrt{x}}{1-\sqrt{x}}dx = \int \frac{(1-x)\left(1-\sqrt{x}\right)}{1-x}dx = \int 1-\sqrt{x}\,dx =$$

$$x + \frac{2x^{3/2}}{3} + C$$

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10)

$$\int 2xe^{3x}\,dx = 2\int xe^{3x}\,dx$$

$$f = x \qquad g' = e^{3x}$$
$$f' = 1 \qquad g = e^{3x}/3$$

$$2\int xe^{3x}\,dx = 2\left[\frac{xe^{3x}}{3} - \int \frac{e^{3x}}{3}\,dx\right] = \frac{2xe^{3x}}{3} - \frac{2e^{3x}}{9} + C$$

15)

$$\int x^2 \ln x\,dx$$

$$f = \ln x \qquad g' = x^2$$
$$f' = \frac{1}{x} \qquad g = \frac{x^3}{3}$$

$$\int x^2 \ln x\,dx = \frac{x^3 \ln x}{3} - \int \frac{x^2}{3}\,dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$$

18)

$$\int \sin^{-1} x\,dx$$

$$f = \sin^{-1} x \qquad g' = 1$$
$$f' = \frac{1}{\sqrt{1-x^2}} \qquad g = x$$

$$\int \sin^{-1} x\,dx = x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

Substitute $u = 1 - x^2 \rightarrow du = -2x\,dx$

$$x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}\,dx = x\sin^{-1} x - \int \frac{du}{-2\sqrt{u}} = x\sin^{-1} x + \sqrt{u} =$$
$$x\sin^{-1} x + \sqrt{1-x^2} + C$$

26)

$$\int x^2 \ln^2 x\, dx$$

$$f = \ln^2 x \qquad g' = x^2$$

$$f' = \frac{2\ln x}{x} \qquad g = \frac{x^3}{3}$$

$$\int x^2 \ln^2 x\, dx = \frac{x^3 \ln^2 x}{3} - \frac{2}{3}\int x^2 \ln x\, dx$$

From Problem 15 $\int x^2 \ln x\, dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$

So

$$\int x^2 \ln^2 x\, dx = \frac{x^3 \ln^2 x}{3} - \frac{2}{3}\left[\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right] =$$

$$\frac{x^3 \ln^2 x}{3} - \frac{2x^3 \ln x}{9} + \frac{2x^3}{27} + C$$

34)

$$\int_0^{\ln 2} xe^x\, dx$$

$$f = x \qquad g' = e^x$$

$$f' = 1 \qquad g = e^x$$

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C$$

$$\int_0^{\ln 2} xe^x\, dx = xe^x - e^x\Big|_0^{\ln 2} = 2\ln 2 - 2 - (0 - 1) = 2\ln 2 - 1$$