Section 7.8 Improper Integrals P. 578 (5,6,7,8,36,37)

5)

$$\int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{a\to\infty}\int_1^{a} \frac{1}{x^2}\,dx = \lim_{a\to\infty}\left[-\frac{1}{x}\right]_1^{a} = \lim_{a\to\infty} -\frac{1}{a} - \frac{-1}{1} =$$

6)

$$\int_0^{\infty} \frac{1}{(1+x)^3}\,dx = \lim_{a\to\infty}\int_0^{a} \frac{1}{(1+x)^3}\,dx = \lim_{a\to\infty}\left[-\frac{1}{(1+x)^2}\right]_0^{a} = \lim_{a\to\infty} -\frac{1}{2(1+a)^2} - \frac{-1}{2} = \frac{1}{2}$$

7)

$$\int_{-\infty}^{0} e^x\,dx = \lim_{a\to-\infty}\int_a^{0} e^x\,dx = \lim_{a\to-\infty}\left[e^x\right]_a^0 = \lim_{a\to\infty} 1 - e^a = 1$$

8)

$$\int_1^{\infty} 2^{-x}\,dx = \lim_{a\to\infty}\int_1^{a} 2^{-x}\,dx = \lim_{a\to\infty}\left[\frac{-2^{-x}}{\ln 2}\right]_1^{a} = \lim_{a\to\infty} \frac{-2^{-a}}{\ln 2} - \frac{-2^{-1}}{\ln 2} = \frac{1}{2\ln 2} = \frac{1}{\ln 4}$$

36)

$$\int_0^{\infty} \frac{1}{\sqrt[3]{x}}\,dx = \int_0^{1} \frac{1}{\sqrt[3]{x}}\,dx + \int_1^{\infty} \frac{1}{\sqrt[3]{x}}\,dx = \lim_{a\to 0}\int_a^{1} \frac{1}{\sqrt[3]{x}}\,dx + \lim_{a\to\infty}\int_1^{a} \frac{1}{\sqrt[3]{x}}\,dx$$

$$\lim_{a\to\infty}\left[\frac{3x^{2/3}}{2}\right]_1^{a} = \frac{3a^{2/3}}{2} - \frac{3}{2} \to \infty$$

So the integral is divergent.

37)

$$\int_1^{2} \frac{1}{\sqrt{x-1}}\,dx = \lim_{a\to 1}\int_a^{2} \frac{1}{\sqrt{x-1}}\,dx = \lim_{a\to 1}\left[2\sqrt{x-1}\right]_a^{2} = \lim_{a\to 1} 2 - 2\sqrt{a-1} = 2$$