Section 6.3 Page 430 (7,8,19,23,28,34,46)

7) First find the intersection of $y = x^2$ and $y = 2 - x^2$

$x^2 = 2 - x^2 \to 2x^2 - 2 = 0 \to x^2 - 1 = 0 \to (x+1)(x-1) = 0 \to x = -1, 1$

The area function is $A(x) = \left((2-x^2) - x^2\right)^2 = 4\left(1-x^2\right)^2$

So the volume is $V = \int_{-1}^{1} A(x)\,dx = 4\int_{-1}^{1} \left(1-x^2\right)^2\,dx$

Notice that the function being integrated is even, so

$$4\int_{-1}^{1} \left(1-x^2\right)^2\,dx = 8\int_{0}^{1} \left(1-x^2\right)^2\,dx = 8\int_{0}^{1} x^4 - 2x^2 + 1\,dx =$$

$$8\left[\frac{x^5}{5} - \frac{2x^3}{3} + x\right]_0^1 = 8\left[\frac{1}{5} - \frac{2}{3} + 1\right] = \frac{64}{15}$$

8) The area function is $A(x) = \left(\sqrt{1-x^2}\right)^2 = 1 - x^2$

So the volume is $V = \int_{-1}^{1} 1 - x^2\,dx$

Notice that the function being integrated is even, so

$$\int_{-1}^{1} 1 - x^2\,dx = 2\int_{0}^{1} 1 - x^2\,dx = 2\left[x - \frac{x^3}{3}\right]_0^1 = 2\left[1 - \frac{1}{3}\right] = \frac{4}{3}$$

19) Using the disk method, the area function is

$A(x) = \pi r^2 = \pi\left(e^{-x}\right)^2 = \pi e^{-2x}$

So the volume is $V = \int_{0}^{\ln 4} \pi e^{-2x}\,dx = \pi\left[\frac{e^{-2x}}{-2}\right]_0^{\ln 4} = -\frac{\pi}{2}\left[e^{-2\ln 4} - e^0\right] = -\frac{\pi}{2}\left[\frac{1}{\left(e^{\ln 4}\right)^2} - 1\right] =$

$$-\frac{\pi}{2}\left[\frac{1}{4^2} - 1\right] = -\frac{\pi}{2}\left[-\frac{15}{16}\right] = \frac{15\pi}{32}$$

23) The area function is $A(x) = \pi\left(\frac{1}{\sqrt[4]{1-x^2}}\right)^2 = \frac{\pi}{\sqrt{1-x^2}}$

So the volume is $V = \int_{0}^{1/2} \frac{\pi}{\sqrt{1-x^2}}\,dx = \pi\left[\arcsin x\right]_0^{1/2} = \pi\left[\frac{\pi}{6} - 0\right] = \frac{\pi^2}{6}$

28) The washer area is $A(x)=\pi\left[\left(\sqrt[4]{x}\right)^2-x^2\right]$

So the volume is $V=\int_0^1 \pi\left[\left(\sqrt[4]{x}\right)^2-x^2\right]dx=\pi\int_0^1 x^{1/2}-x^2\,dx=\pi\left[\frac{2x^{3/2}}{3}-\frac{x^3}{3}\right]_0^1=$

$\pi\left[\frac{2}{3}-\frac{1}{3}\right]=\frac{\pi}{3}$

34) Find the points of intersection by setting $|x|=2-x^2$ giving $x=-1,1$

The area function is $A(x)=\pi\left(\left(2-x^2\right)^2-\left(|x|\right)^2\right)=\pi\left[x^4-4x^2+4-x^2\right]=\pi\left[x^4-5x^2+4\right]$

So the volume is $V=\int_{-1}^{1}\pi\left[x^4-5x^2+4\right]dx$

Notice that the function being integrated is even, so

$\int_{-1}^{1}\pi\left[x^4-5x^2+4\right]dx=2\pi\int_0^1 x^4-5x^2+4\,dx=$

$2\pi\left[\frac{x^5}{5}-\frac{5x^3}{3}+4x\right]_0^1=2\pi\left[\frac{1}{5}-\frac{5}{3}+4\right]=\frac{76\pi}{15}$

46) You can either use the washer method or instead use the disk method and then subtract the volume of the internal cylinder. I'll use the washer method here.

The area function is $A(x)=\pi R^2-\pi r^2=\pi 4^2-\pi(4-x)^2=\pi\left(16-\left(4-y^2\right)^2\right)$

So the volume is $V=\pi\int_0^2 16-\left(4-y^2\right)^2\,dy=\pi\int_0^2 8y^2-y^4dy=\pi\left[\frac{8y^3}{3}-\frac{y^5}{5}\right]_0^2=$

$\pi\left[\frac{64}{3}-\frac{32}{5}\right]=\frac{224\pi}{15}$