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21)

Notice that as θ goes from $-\frac{\pi}{2} \to 0$ that $\cos\theta$ goes from $0 \to 1$

Likewise as θ goes from $0 \to \frac{\pi}{2}$ that $\cos\theta$ goes from $1 \to 0$

All other values of θ are undefined.

So we have the area $A = \int_{-\pi/2}^{\pi/2} \frac{1}{2}\left(\sqrt{\cos\theta}\right)^2 d\theta = \int_{-\pi/2}^{\pi/2} \frac{1}{2}\cos\theta\, d\theta = \frac{1}{2}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} = \frac{1}{2}\left[1 - {}^{-}1\right] = 1$

22) For similar reasons to 21) The interval to integrate on is $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

So we have the area $A = \int_{-\pi/4}^{\pi/4} \frac{1}{2}\left(\sqrt{\cos 2\theta}\right)^2 d\theta = \int_{-\pi/4}^{\pi/4} \frac{1}{2}\cos 2\theta\, d\theta = \frac{1}{4}\left[\sin 2\theta\right]_{-\pi/4}^{\pi/4} = \frac{1}{4}\left[1 - {}^{-}1\right] = \frac{1}{2}$

23) As θ goes from $0 \to \pi$ $8\sin\theta$ goes from $0 \to 8 \to 0$ completing the circle.

So we have $A = \int_0^{\pi} \frac{1}{2}(8\sin\theta)^2 d\theta = \frac{64}{2}\int_0^{\pi} \sin^2\theta\, d\theta = 32\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta =$

$16\int_0^{\pi} 1 - \cos 2\theta\, d\theta = 16\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} = 16\left[(\pi - 0) - (0 - 0)\right] = 16\pi$

24) As θ goes from $0 \to 2\pi$ that $4 + 4\sin\theta$ goes from $4 \to 8 \to 4 \to 0 \to 4$ completing the cartoid.

So we have

$A = \int_0^{2\pi} \frac{1}{2}(4 + 4\sin\theta)^2 d\theta = \int_0^{2\pi} 8 + 16\sin\theta + 8\sin^2\theta\, d\theta = \int_0^{2\pi} 8 + 16\sin\theta + 8\cdot\frac{1-\cos 2\theta}{2}\, d\theta =$

$\int_0^{2\pi} 12 + 16\sin\theta - 4\cos 2\theta\, d\theta = \left[12\theta - 16\cos\theta - 2\sin 2\theta\right]_0^{2\pi} =$

$\left[24\pi - 16 - 0 - (0 - 16 - 0)\right] = 24\pi$