Section 7.9 Differential Equations

1)

$$y''+5y'+4y=0 \to x^2+5x+4=0 \to (x+1)(x+4)=0$$

$$y(x)=Ae^{-x}+Be^{-4x}$$

$$y'(x)=-Ae^{-x}-4Be^{-4x}$$

a)

$$y(0)=-1 \quad y'(0)=1$$

$$y(0)=A+B=-1$$

$$y'(0)=-A-4B=1$$

$$A=-1 \quad B=0$$

$$y(x)=-e^{-x}$$

2)

$$y''+2y'+4y=0 \to x^2+2x+4=0 \to (x+2)(x+2)=0$$

$$y(x)=Ae^{-2x}+Bxe^{-2x}$$

$$y'(x)=-2Ae^{-x}+B\left(-2xe^{-2x}+e^{-2x}\right)$$

a)

$$y(0)=1 \quad y'(0)=0$$

$$y(0)=A=1$$

$$y'(0)=-2A+B=0$$

$$A=1 \quad B=2$$

$$y(x)=e^{-2x}+2xe^{-2x}$$

b)

$$y(0)=-1 \quad y'(0)=-1$$

$$y(0)=A=-1$$

$$y'(0)=-2A+B=-1$$

$$A=-1 \quad B=-3$$

$$y(x)=-e^{-2x}-3xe^{-2x}$$

3)

$$y''+4y'+5y=0 \rightarrow x^2+4x+5=0 \rightarrow x=-2\pm i$$

$$y(x)=Ae^{(-2+i)x}+Be^{(-2-i)x}=e^{-2x}(a\sin x+b\cos x)$$

$$y'(x)=e^{-2x}(a\cos x-b\sin x)+-2e^{-2x}(a\sin x+b\cos x)$$

a)

$$y(0)=1/2 \quad y'(0)=1$$

$$y(0)=b=1/2$$

$$y'(0)=a-2b=1$$

$$a=2$$

$$y(x)=e^{-2x}\left(2\sin x+\frac{\cos x}{2}\right)$$

b)

$$y(0)=-1 \quad y'(0)=-3$$

$$y(0)=b=-1$$

$$y'(0)=a-2b=1$$

$$a=5$$

$$y(x)=e^{-2x}(-5\sin x+\cos x)$$

4)

$$y''+4y'+4y=0$$

$$x^2+4x+4=0$$

$$(x+2)^2=0$$

$$Dim=2$$

$$Basis=\left\{e^{-2x},xe^{-2x}\right\}$$

$$y(x)=Ae^{-2x}+Bxe^{-2x}$$

5)

$y''' - 3y'' + 3y' - y = 0$

$x^3 - 3x^2 + 3x - 1 = 0$

$(x-1)^3 = 0$

$Dim = 1$

$Basis = \{e^x, xe^x, x^2e^x\}$

$y(x) = Ae^x + Bxe^x + Cx^2e^x$

6)

$y^2 = kx^3 \to k = y^2 / x^3$

Differentiating

$$2y\frac{dy}{dx} = 3kx^2$$

$$\frac{dy}{dx} = \frac{3kx^2}{2y}$$

$$m = -\frac{2y}{3kx^2} = -\frac{2x}{3y}$$

$$\frac{dy}{dx} = -\frac{2x}{3y}$$

$$\int y\,dy = \int -\frac{2}{3}x\,dx$$

$$y^2 = -\frac{x^2}{3} + C$$

$$\frac{y^2}{C} + \frac{x^2}{3C} = 1$$

A family of ellipses

7)

$$y = \frac{k}{x} \rightarrow k = xy$$

$$\frac{dy}{dx} = -\frac{k}{x^2}$$

$$m = \frac{x^2}{k} = \frac{x}{y}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\frac{y^2}{A} - \frac{x^2}{A} = 1$$

A family of hyperbolas