M1B/Schoenbrun Section 5.5 Substitution

Evaluate using substitution

1) $\int_0^3 x\sqrt{1+x}\,dx$

$u = 1+x \quad x = u-1$
$du = dx$

$$\int_0^3 x\sqrt{1+x}\,dx = \int_1^4 (u-1)\sqrt{u}\,du = \int_1^4 u^{3/2} - u^{1/2}du = \left[\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}\right]_1^4 =$$

$$2\left[\left(\frac{32}{5} - \frac{8}{3}\right) - \left(\frac{1}{5} - \frac{1}{3}\right)\right] = 2\left[\frac{96-40}{15} - \frac{3-5}{15}\right] = \frac{2\cdot 58}{15} = \frac{116}{15}$$

2) $\int x^2\left(x^3+5\right)^9 dx$

$u = x^3 + 5$

$du = 3x^2dx \quad x^2dx = \frac{du}{3}$

$$\int x^2\left(x^3+5\right)^9 dx = \frac{1}{3}\int\left(x^3+5\right)^9 3x^2\,dx = \frac{1}{3}\int u^9\,du = \frac{u^{10}}{30} + C$$

3) $\int \frac{1-e^x}{1+e^x}dx = \int \frac{1+e^x}{1+e^x} - \frac{2e^x}{1+e^x}dx = \int 1\,dx - \int \frac{2e^x}{1+e^x}dx = x - \int \frac{2e^x}{1+e^x}dx$

$u = 1+e^x$
$du = e^x dx$

$$x - 2\int \frac{e^x}{1+e^x}dx = x - 2\int \frac{du}{u} = x - 2\ln|u| + C = x - 2\ln\left|1+e^x\right| + C$$

4) $\int x^3 \cos\left(x^4+1\right)dx$

$u = x^4 + 1$

$du = 4x^3 dx$

$x^3 dx = \frac{du}{4}$

$$\int x^3 \cos\left(x^4+1\right)dx = \int \cos\left(x^4+1\right)x^3\, dx = \int \frac{\cos u\, du}{4} = \frac{\sin u}{4} + C = \frac{\sin\left(x^4+1\right)}{4} + C$$

5) $\int x(1+x)^{1/3}\, dx$

$u = 1 + x$

$du = dx$

$x = u - 1$

$$\int x(1+x)^{1/3}\, dx = \int (u-1)u^{1/3} du = \int u^{4/3} - u^{1/3} du = \frac{3u^{7/3}}{7} - \frac{3u^{4/3}}{4} + C = \frac{3(1+x)^{7/3}}{7} - \frac{3(1+x)^{4/3}}{4} + C$$

6) $\int_0^1 2x^2 (4x+1)^{-5/2}\, dx$

$u = 4x + 1$

$du = 4dx$

$dx = \frac{du}{4}$

$x = \frac{u-1}{4}$

$$\int_0^1 2x^2(4x+1)^{-5/2}\, dx = \int_1^5 \frac{2(u-1)^2}{16} \cdot \frac{u^{-5/2} du}{4} = \frac{1}{32}\int_1^5 u^{-1/2} - 2u^{-3/2} + u^{-5/2} du =$$

$$\frac{1}{32}\left[2u^{1/2} + 4u^{-1/2} - \frac{2}{3}u^{-3/2}\right]_1^5 = \frac{1}{32}\left[\left(2\sqrt{5} + \frac{4}{\sqrt{5}} - \frac{2}{3\left(5^{3/2}\right)}\right) - \left(2 + 4 - \frac{2}{3}\right)\right] =$$

$$\frac{1}{32}\left[2\sqrt{5} + \frac{4}{\sqrt{5}} - \frac{2}{3\left(5^{3/2}\right)} - \frac{16}{3}\right]$$

It’s not pretty, but it’s exact, :-)