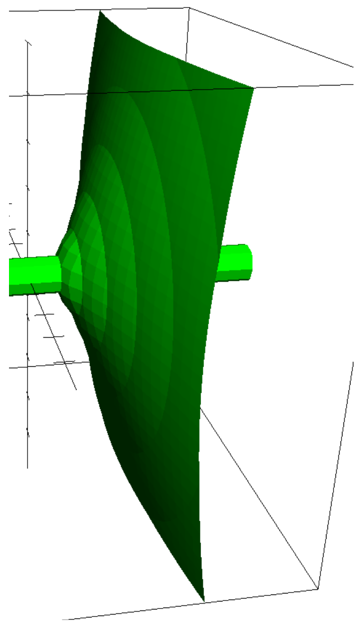
Section 6.4 Volumes by Shells Worksheet Solutions

1) Find the volume generated by revolving $y = x^2$ around the x-axis on the interval [0,1].

Shell Method



Integrate along the y axis as the radius of shells.

The height of the shells are $f(y) = 1 - x = 1 - \sqrt{y}$

Using the shell formula $V = \int_0^1 2\pi y f(y)\,dy = \int_0^1 2\pi y\left(1-\sqrt{y}\right)dy = 2\pi\int_0^1 y - y^{3/2}\,dy = 2\pi\left[\frac{y^2}{2} - \frac{2y^{5/2}}{5}\right]_0^1 =$

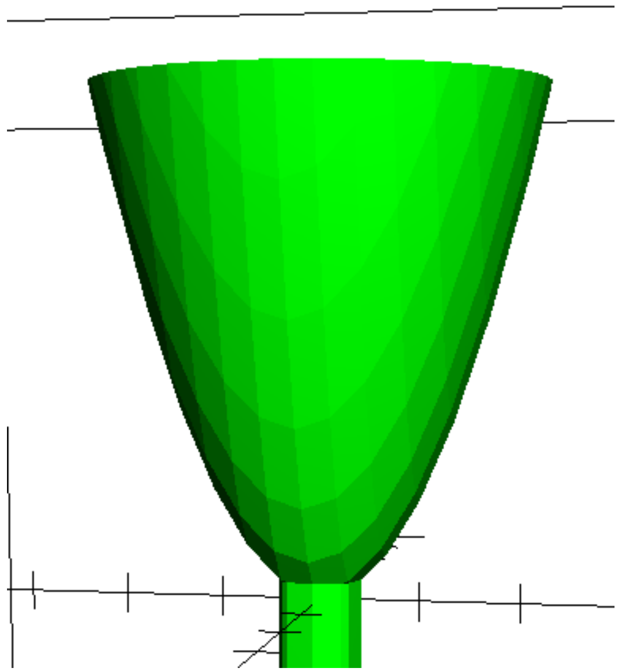
$2\pi\left[\frac{1}{2} - \frac{2}{5}\right] = 2\pi\frac{1}{10} = \frac{\pi}{5}$

Check using the Disk method:

$r = y = x^2$

$V = \int_0^1 \pi r^2\,dx = \int_0^1 \pi\left(x^2\right)^2 dx = \int_0^1 \pi x^4\,dx = \pi\left[\frac{x^5}{5}\right]_0^1 = \frac{\pi}{5}$

Repeat problem 1 but revolve $y = x^2$ around the y axis.

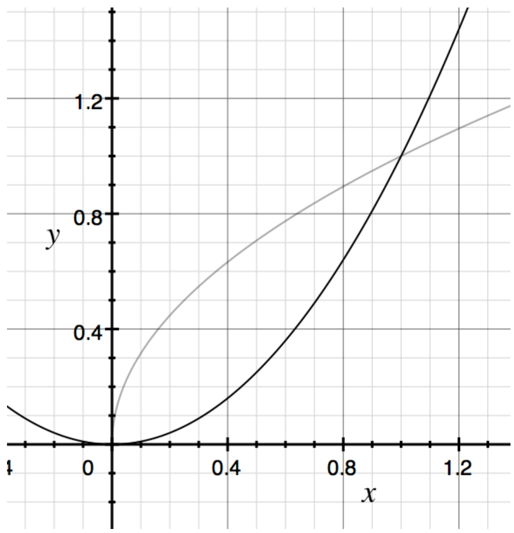


Integrate along the x axis.

The height of the shells are $f(x) = y = x^2$

$$V = \int_0^1 2\pi x \cdot x^2 \, dx = 2\pi \int_0^1 x^3 \, dx = 2\pi \left[\frac{x^4}{4} \right]_0^1 = \frac{\pi}{2}$$

2) Use the shell method to find the volume generated by revolving the region between $y = \sqrt{x}$ and $y = x^2$ on the interval [0,1] around the x-axis.



Integrate along the y axis as the radius of shells.

The height of the shells are $f(y) = \sqrt{y} - y^2$
Using the shell formula

$$V = \int_0^1 2\pi y f(y) dy = \int_0^1 2\pi y\left(\sqrt{y} - y^2\right) dy = 2\pi \int_0^1 y^{3/2} - y^3 \, dy = 2\pi \left[\frac{2y^{5/2}}{5} - \frac{y^3}{3}\right]_0^1 =$$

$$2\pi\left[\frac{2}{5} - \frac{1}{3}\right] = \frac{2\pi}{15}$$