Lesson Plan 1 - First Day of Class

1) Take attendance2) Introduce yourself
    Background, Degrees, Years teaching, 3nd quarter at Foothill
3) Pass out green sheet
4) Mention Website schoenbrun.com/foothill
5) Office hours 1/2 hour before and after class

Go over green sheet
6) You need for the class
    Textbook - Mention Student Website: www.tbookswap.com
    Graphic Calculator Ti83/84
    Need to do homework, and need be here for tests, quizzes and final
7) How many took Calculus 1a? Last quarter?
8) How many first time Calculus students?

This is a difficult class. Be prepared to work hard.
Please come on time.
If you come in late, need to excuse yourself or leave early, please try to not be disruptive.
This class is long.
We will have a short break in the middle.
Feel free to let me know if I forget.
I am fairly loose about when you turn in your homework.
Usually I will answer questions at the beginning of class before collecting it.
Homework is important to get practice with the skills you need to learn in this class.
Do not be fooled by the low percentage is applies to your grade.
Without practice you will have difficulty on tests and quizzes.
Help is available in the STEM center.
I may assign some online homework.

What you need to know so succeed in this class
A) High School Algebra
B) High School Geometry
C) High School Pre-Calculus, Functions and Trigonometry
D) First quarter Calculus
Limits, L'Hôpital's rule
Continuity
Derivatives of
    1) Polynomials
    2) Exponential functions and logs
    3) Trig and inverse trig Functions
Product, Quotient and Chain rules
Derivatives using the inverse of a function
Anti-Derivatives (check whether this was covered in 1B)
Explain EXACT vs. computed answers......
Any questions?

In Calc 1 we learned how to find the slope of a tangent to a function, or an instantaneous rate of a function by finding the derivative of that function.

In this class we will be learning how to calculate the area under a function on sum interval, and the relationship this has to differentiation.

We will start by defining a Riemann sum which are named after Bernhard Riemann, a German mathematician who lived 1826-1866:

What is a Riemann sum, a formal definition.

We take a function $f(x)$ defined on an interval $[a,b]$

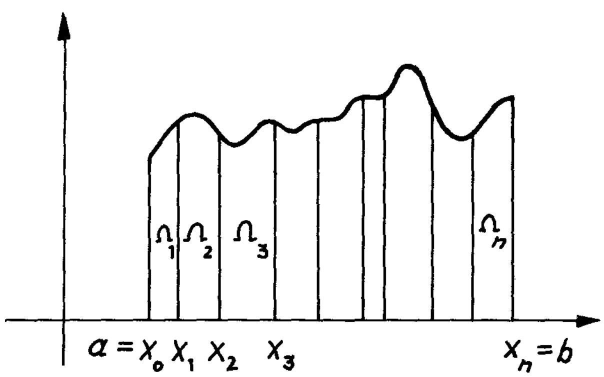
We partition this interval using n **grid points** $\{a = x_0, x_1, ..., x_n = b\}$ where $a = x_0 < x_1 < ... < x_n = b$.

This defines n intervals $[x_{i-1}, x_i]$.

Finally we create an area sum as follows:

$\sum_{i=1}^{n} f(x_i^*)\Delta x_i$ where

$x_i^* \in [x_{i-1}, x_i]$ and $\Delta x = x_i - x_{i-1}$



Digression:

$$\sum_{i=1}^{5} i = 1+2+3+4+5$$

$$\sum_{i=1}^{5} i^2 = 1+4+9+16+25$$

Note that i is a dummy variable

Also note that $x \in [a,b]$ means that x is a point in the interval $[a,b]$
$\in$ is the set theory symbol meaning an object is a member of a set.

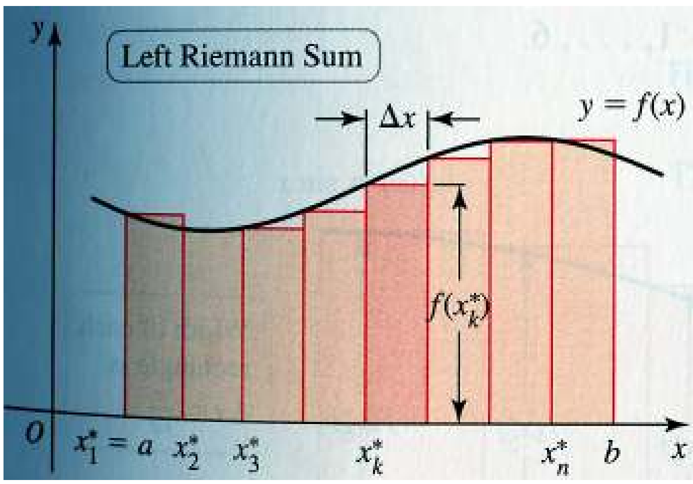
Side Note: The book deals mainly with Riemann sums where Δx is a constant.

$$\Delta x = \frac{b-a}{n}$$

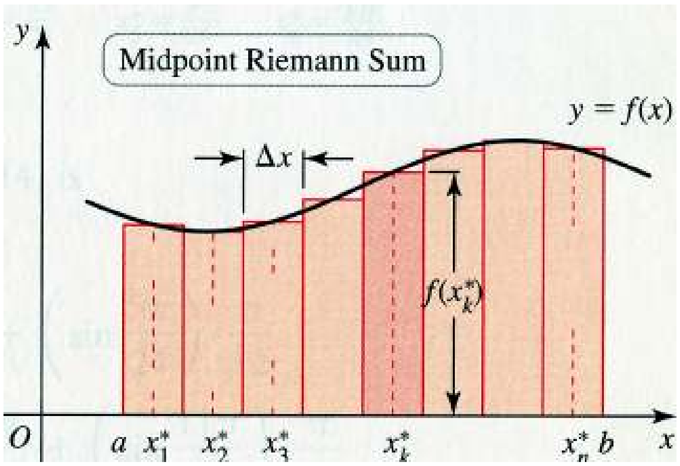
This is called a **regular partition** in contrast to a **general partition** where the sizes of the $\Delta x's$ can vary.

With x_i^* set as follows you get

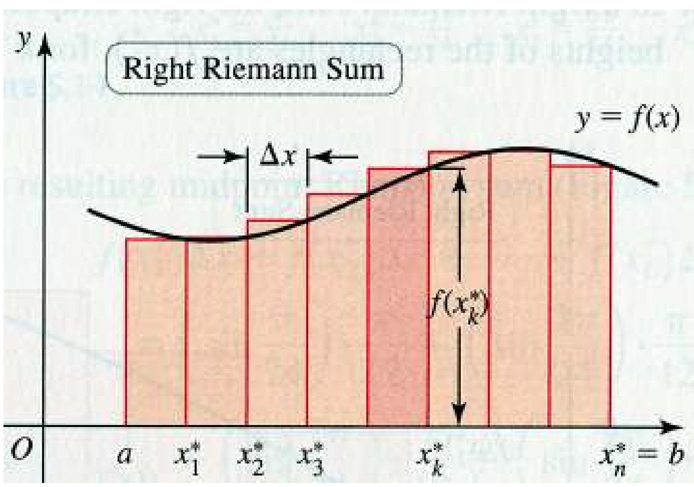
$x_i^* = x_{i-1}$ (A left Riemann sum



$x_i^* = \frac{x_{i-1} + x_x}{2}$ (A mid-point Riemann sum



$x_i^* = x_i$ (A right Riemann sum



IMPORTANT!

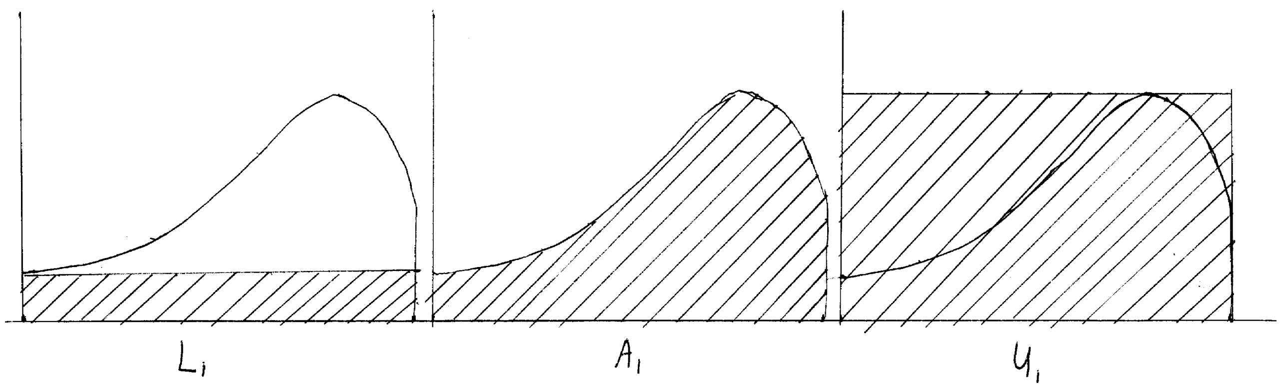
While these three are all Riemann sums, they are not useful for the main reason we are studying Riemann sums which is to define what a Riemann Integral is.

Instead we define Lower and Upper Riemann sums as follows:

For a lower Riemann sum x_i^* is the least value in the interval $\left[x_{i-1}, x_i\right]$

For an upper Riemann sum x_i^* is the greatest value in the interval $\left[x_{i-1}, x_i\right]$

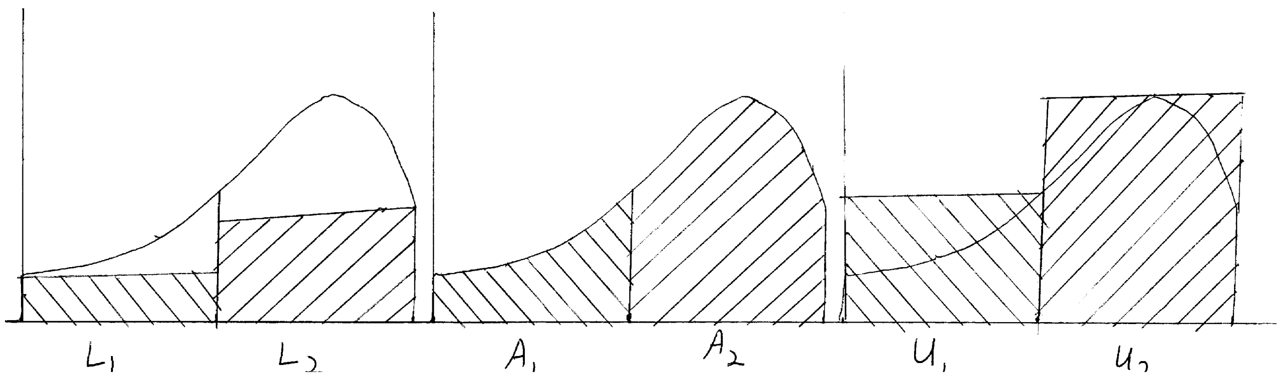
Let's start with Upper and Lower Riemann sums for some function where n=1



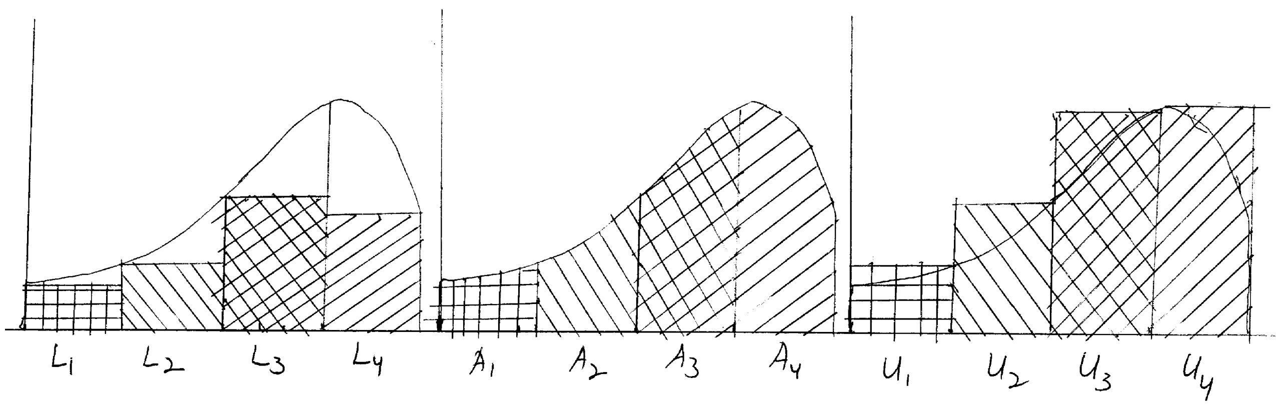
Note that by the way we construct these sums $L_1 \leq A \leq U_1$

You might want to note that if $f(x) = c$ a constant then $L_1 = A = U_1$

Increasing n to 2 we see



and to 4



Note that in all cases

$L_i \le A_i \le U_i$

And therefore $\sum_{i=1}^{n} L_i \le A \le \sum_{i=1}^{n} U_i$

We are now ready to give the definition of a Riemann integral.

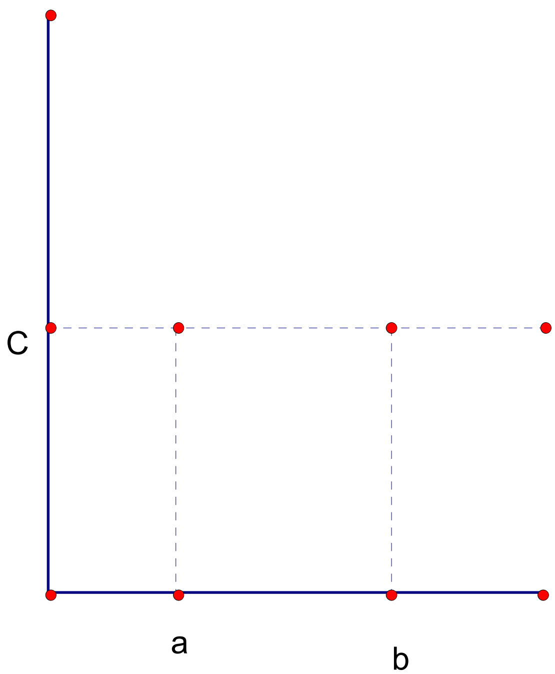
If there is a limit $\lim_{n\to\infty} \sum_{i=1}^{n} L_i = A_l$ and also $\lim_{n\to\infty} \sum_{i=1}^{n} U_i = A_u$ and $A_l = A_u = A$

then we say the function $f(x)$ is Riemannian integrable and we indicate this as follows:

$$A = \int_a^b f(x)dx$$

We now have a somewhat abstract definition of an integral, however we don't know much yet about how to calculate one. To get a feel for where this is going we look at a few simple examples.

$$\int_a^b C\,dx$$

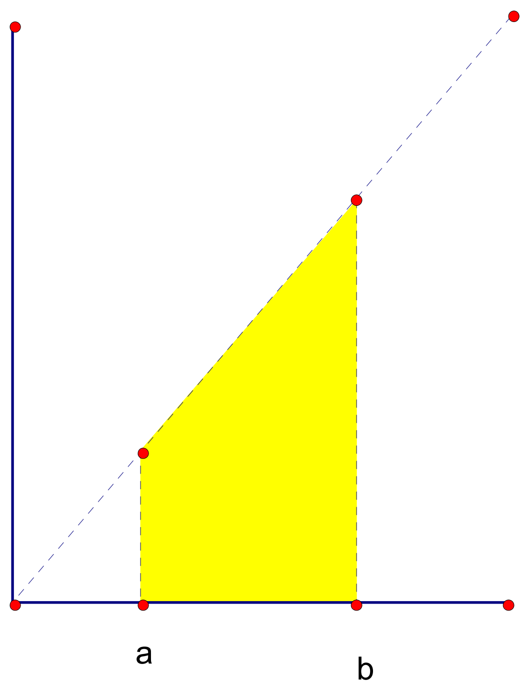


Note that for any partition of $[a,b]$ that $\sum_{i=1}^{n} L_i = C(b-a) = \sum_{i=1}^{n} U_i$

This is of course the formula for the area of a rectangle.
Note that we can write this as $A = C(b-a)$ which we can write $A = Cx\Big|_a^b$

As a side note: $\frac{d}{dx}Cx = C$

Take as a second example: $\displaystyle\int_a^b x\ dx$



Note that this can be calculated as the difference in areas of the right triangles:

$$\frac{1}{2}b \cdot b - \frac{1}{2}a \cdot a = \frac{b^2}{2} - \frac{a^2}{2}$$

We could also write this in a form similar to the last example as $A = \left.\frac{x^2}{2}\right|_a^b$

Also note that $\frac{d}{dx}\frac{x^2}{2} = x$

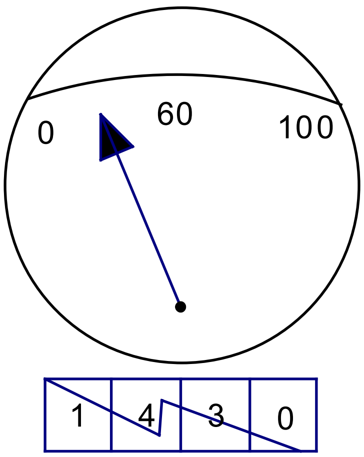
This suggests that the an area function $F(x)$ where $A = F(b) - F(a) = F(x)\Big|_a^b$

has the property that when $A = \int_a^b f(x)\,dx = F(x)\Big|_a^b$ that

$\frac{d}{dx}F(x) = f(x)$, That is $F(x)$ is the **anti-derivative** of $f(x)$. We will study this more at a later time.

A speed distance Problem:

Imagine you are traveling in vehicle on a straight road and you can read the speedometer and a clock, but the odometer is broken. Further imagine that there are no road signs to tell you how far you have traveled.



You might figure out the distance you are traveling as follows:

Pick a time interval, call it Δt, maybe 60 seconds.

During the interval write down the minimum speed of the car v_i
and the maximum speed of the car V_i

Doing this you two sets of distances:

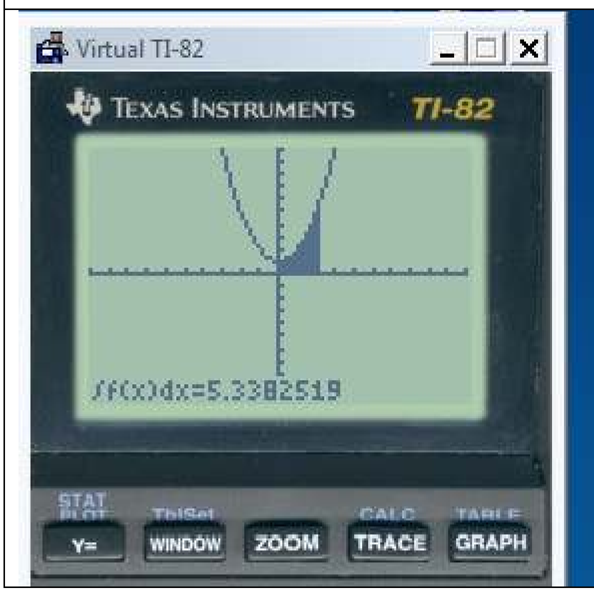
$d_i = v_i \Delta t \leq V_i \Delta t = D_i$

Summing these distances $D = vt$ you get upper and lower limits on your total distance:
$\sum_{i=1}^{n} d_i \leq X \leq \sum_{i=1}^{n} D_i$

Now if you measure in smaller and smaller time intervals n grows larger and larger.
Taking the limit as $n \longrightarrow \infty$ both the lower and upper bounds will converge to the actual distance.

Note this is exactly the same mathematical operation we performed when calculating area using a Riemann sum.

Mechanical calculation of a definite Integral using a Ti-8x



Note that this computerl method also produces an approximation.

Next class we will go on to learn how to calculate a definite integral exactly.