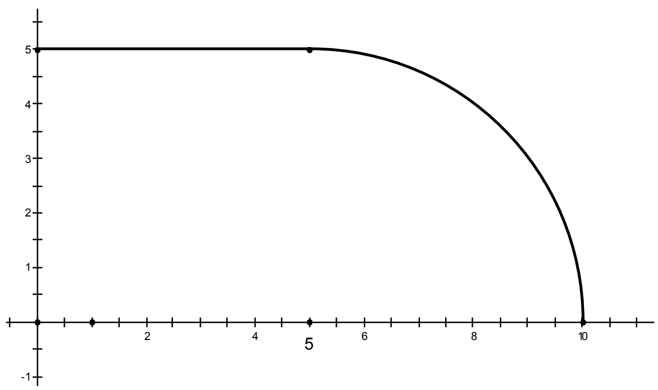
Foothill College Math 1B - MidTerm sections 5.1-5.5, 6.2, 6.3, 6.5 Mitchell Schoenbrun

1)[5] The function $f(x)$ is defined as:

$$f(x)=\begin{cases} 0\le x\le 5 & f(x)=5 \\ 5\le x\le 10 & f(x)=\sqrt{25-(x-5)^2} \end{cases}$$



Find $\displaystyle\int_0^{10} f(x)\,dx$ EXACTLY ____________________________

Note that the area consists of a 5x5 rectangle and 1/4 of a circle with radius 5.

$$\int_0^{10} f(x)\,dx = 25+25\pi/4$$

2)[5] Evaluate the sum

$$\sum_{k=1}^{65} 1 = 1+1+1+\ldots.+1 = 65$$

3)[5] Evaluate the indefinite integral

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

4)[5] Evaluate the definite integral EXACTLY

$$\int_{-2}^{2} x^5 - \sin(x)\,dx = \underline{\hspace{6cm}}$$

Hint: (What kind of function is $x^5 - \sin(x)$?

The function is odd so the integral is ZERO

5)[5] Use your CALCULATOR to evaluate the definite integral APPROXIMATELY

$$\int_{0}^{\pi/6} e^{-x^4}\,dx = .5158892$$

6)[10] Use your CALCULATOR to find the APPROXIMATE area between the curves $y = \ln(x)$ and $y = x^2 - 2$. Show at least 4 decimal places

First put the function ln(x)-(x^2-2) into your calculator and find it's zeros at

.13793483 and 1.5644623

Then use the integrate function to find the value 1.1244479

7) [10]

$$F(x) = \int_{x^3}^{x^2} \sin(t)\,dt$$

Find $\frac{dF}{dx} = \underline{\hspace{5cm}}$

$$\int_{x^3}^{x^2} \sin(t)\,dt = \int_{x^3}^{A} \sin(t)\,dt + \int_{A}^{x^2} \sin(t)\,dt = \int_{A}^{x^2} \sin(t)\,dt - \int_{A}^{x^2} \sin(t)\,dt =$$

$$2x\sin(x^2) - 3x^2\sin(x^2) = x\left[2\sin(x^2) - 3x\sin(x^2)\right]$$

8)[10]

Find the $\overline{f}$, the AVERAGE value of $f(x)$ on the interval $[0,2]$ where

$$f(x)=e^x$$

$$Avg=\frac{\int_0^2 e^x dx}{2-0}=\frac{e^x\Big|_0^2}{2}=\frac{e^2-1}{2}$$

9)[5] For the above function and interval, find the value of c where $f(c)=\overline{f}$

$$e^c=\frac{e^2-1}{2}\rightarrow c=\ln\left(\frac{e^2-1}{2}\right)$$

10)[10] Using the substitution $u=x^3+1$ do a change of variables on the definite integral

$$\int_{-1}^{1} x^2\sqrt{x^3+1}\,dx=\int_{\_\_}^{\_\_}\_\_\_\_\_\_\_\_\_\_\,du$$

$$u=x^3+1$$

$$du=3x^2dx$$

$$\frac{du}{3}=x^2dx$$

$$u=1^3+1=2$$

$$u=(-1)^3+1=0$$

$$\int_{-1}^{1} x^2\sqrt{x^3+1}\,dx=\int_{\_0\_}^{\_2\_}\frac{\sqrt{u}}{3}\,du$$

11)[10] What is the EXACT area between the curves $2\sin(x)$ and $-\sin(x)$ on the interval $[0,2\pi]$

Area = $\int_0^{2\pi}\left|2\sin x-{}^{-}\sin x\right|dx=2\int_0^{\pi}3\sin x\,dx=6\left[-\cos x\right]_0^{\pi}=6\left[-(-1)-{}^{-}1\right]=$ 12

12)[10] Find the EXACT volume created by spinning the curve $y = x^{3/4}$ on the interval
$[1,4]$ around the x - axis.

Volume = $\int_1^4 \pi\left(x^{3/4}\right)^2 dx = \pi\int_1^4 x^{3/2}dx = \pi\left[\frac{x^{5/2}}{5/2}\right]_1^4 = \frac{2\pi}{5}[32-1] = 62\pi/3$

10] Find the length of the curve $y = \frac{e^x + e^{-x}}{2}$ on the interval $[0,2]$

$$\int_0^2 \sqrt{1+\left(\frac{d}{dx}\frac{e^x+e^{-x}}{2}\right)^2}\,dx = \int_0^2 \sqrt{1+\left(\frac{e^x-e^{-x}}{2}\right)^2}\,dx = \int_0^2 \sqrt{\left(\frac{e^x+e^{-x}}{2}\right)^2}\,dx$$

$$= \int_0^2 \frac{e^x+e^{-x}}{2}\,dx = \left[\frac{e^x-e^{-x}}{2}\right]_0^2 = \frac{e^2-e^{-2}}{2}$$

Now that you know about hyperbolic functions, you do this with a little less effort:

$$\int_0^2 \sqrt{1+\left(\frac{d}{dx}\cosh\right)^2}\,dx = \int_0^2 \sqrt{1+\sinh^2 x}\,dx = \int_0^2 \sqrt{\cosh^2 x}\,dx$$

$$= \int_0^2 \cosh x\,dx = \left[\sinh x\right]_0^2 = \sinh 2$$