M1B/Schoenbrun Linear Differential Equations

Solve the differential equations with the stated conditions:

1) $y''+5y'+4y=0$ where $y(0)=-1$ and $y'(0)=1$

$r^2+5r+4=0 \rightarrow (r+4)(r+1)=0 \rightarrow r=-4,-1$

$y(x)=Ae^{-x}+Be^{-4x}$

$y'(x)=-Ae^{-x}+-4Be^{-4x}$

$y(0)=A+B=-1$

$y'(0)=-A-4B=1$

$-3B=0 \rightarrow B=0 \rightarrow A=-1$

$y(x)=-e^{-x}$

2) $y''+2y'+y=0$ where a) $y(0)=1$ and $y'(0)=0$
and b) $y(0)=-1$ and $y'(0)=-1$

$r^2+2r+1=0 \rightarrow (r+1)^2=0 \rightarrow r=-1$

$y(x)=Ae^{-x}+Bxe^{-x}$

$y'(x)=-Ae^{-x}+B\left[-xe^{-x}+e^{-x}\right]$

a) $y(0)=A=1$

$y'(0)=-A+B=0 \rightarrow B=1$

$y(x)=e^{-x}+xe^{-x}$

b) $y(0)=A=-1$

$y'(0)=-A+B=-1 \rightarrow B=0$

$y(x)=-e^{-x}$

3) $y''+4y'+5y=0$ where a) $y(0)=\frac{1}{2}$ and $y'(0)=1$ and b) $y(0)=-1$ and $y'(0)=-3$

From problem 1,

$$y(x)=Ae^{-x}+Be^{-4x}$$
$$y'(x)=-Ae^{-x}+-4Be^{-4x}$$

a)

$$y(0)=A+B=\frac{1}{2}$$
$$y'(0)=-A-4B=1$$
$$-3B=\frac{3}{2}\rightarrow B=-\frac{1}{2}\rightarrow A=1$$
$$y(x)=e^{-x}-\frac{1}{2}e^{-4x}$$

b)

$$y(0)=A+B=-1$$
$$y'(0)=-A-4B=-3$$
$$-3B=-4\rightarrow B=\frac{4}{3}\rightarrow A=-\frac{7}{3}$$
$$y(x)=-\frac{7}{4}e^{-x}+\frac{4}{3}e^{-4x}$$