Lesson Plan 1 - First Day of Class

1) Take attendance, any new students?
2) Introduce yourself
    Background, Degrees, Years teaching, 2nd quarter at Foothill
3) Pass out green sheet
4) Mention Website schoenbrun.com/foothill
5) Office hours 1/2 hour before and after class

Go over green sheet
6) You need for the class
    Textbook
    Graphic Calculator Ti83/84
    Need to do homework, and need be here for tests, quizzes and final
7) How many took Calculus 1a?   Last quarter?
8) How many first time Calculus students?

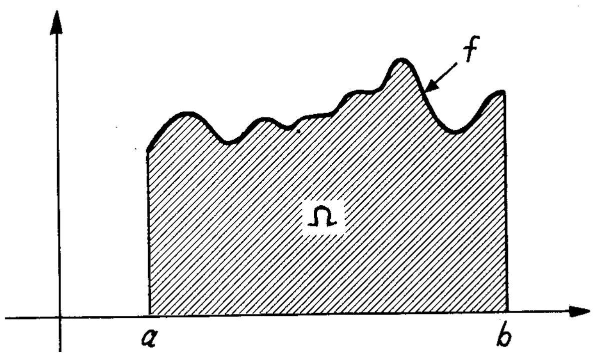
This is a difficult class. Be prepared to work hard.
Please come on time.
If you come in late, need to excuse yourself or leave early, please try to not be disruptive.
This class is long.
We will have a short break in the middle.
Feel free to let me know if I forget.
I am fairly loose about when you turn in your homework.
Usually I will answer questions at the beginning of class before collecting it.
Homework is important to get practice with the skills you need to learn in this class.
Do not be fooled by the low percentage is applies to your grade.
Without practice you will have difficulty on tests and quizzes.
Help is available in the PSME center

What you need to know for this class
A) High School Algebra
B) High School Geometry
C) High School Pre-Calculus, Functions and Trigonometry
D) First quarter Calculus
Limits, L'Hôpital's rule
Continuity
Derivatives of
    1) Polynomials
    2) Exponential functions and logs
    3) Trig and inverse trig Functions
Product, Quotient and Chain rules
Derivatives using the inverse of a function
Anti-Derivatives (check whether this was covered in 1B)
Explain EXACT vs. computed answers......
Any questions?

An expanded idea of Area

Previously we found areas in specific limited contexts.

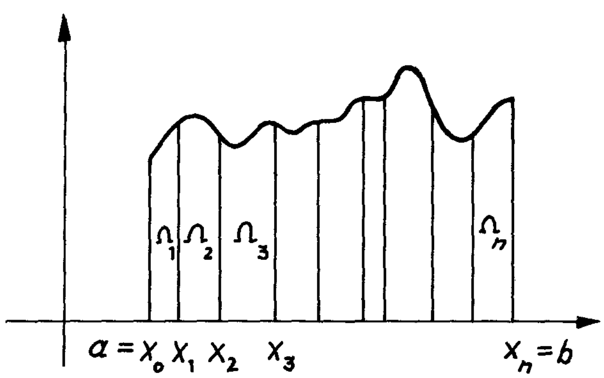
Triangles, rectangles, polygons, circles.
In this class we will learn to calculate areas in a more general way.



Given some general function *f* we want to know what the area underneath the function is between two *x* coordinates *a* and *b*, which we call Ω

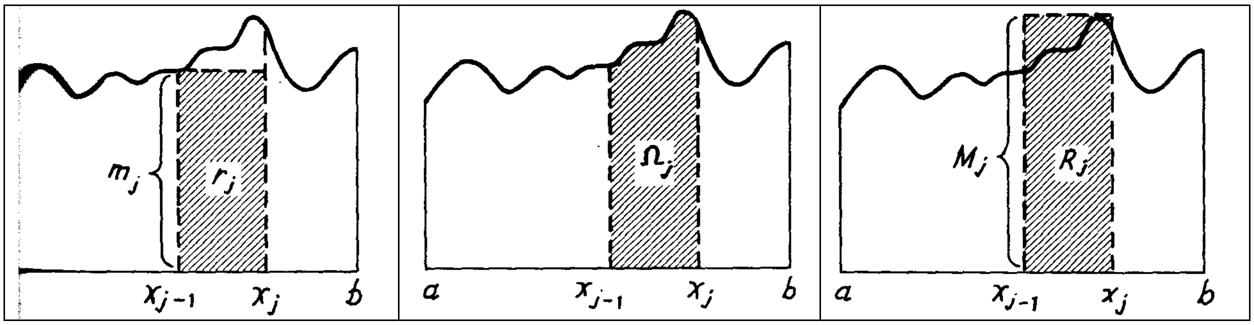
We break the interval [a,b] into a "Partition" P, which is a set of points x_i such that

$P = \{x_0, x_1, ..., x_n\}$ where $x_o = a$, $x_n = b$ and $x_0 < x_1 < ... < x_n$



Calling each sub area in the interval $\left[x_{j-1}, x_j\right]$, Ω_j

For each sub area between the interval $\left[x_{j-1}, x_j\right]$ we determine
minimum height of the function $= m_j$
maximum height of the function = M_j
and therefore we can calculate areas $r_j = m_j\left(x_j - x_{j-1}\right) \le \Omega_j \le R_j = M_j\left(x_j - x_{j-1}\right)$



If we take a sum over these sub areas we have

$$\sum_{j=1}^{n} r_i \le \Omega \le \sum_{j=1}^{n} R_i$$

The sums are called Riemann sums after Bernhard Riemann, a German mathematician who lived 1826-1866:

Let left sum is called the Lower Riemann sum $L_f\left(P\right)$
and the right, the Upper Riemann sum $U_f\left(P\right)$.
We define the area Ω to be the value that is true for all partitions *P* of the interval $\left[a, b\right]$.

A note about the text:
The text book uses a slight less rigorous definition of area. Instead of using a general partition, the book divides the interval into equal spaced intervals $\Delta x = \frac{(b-a)}{n}$ and chooses a "sample" point within each interval calling it x_j^*.

It then defines the area as: $\lim_{n \to \infty} \sum_{i=1}^{n} f\left(x_i^*\right) \Delta x$.

For purposes of this course, this is an adequate definition but will not be used if you continue on to a first year college Analysis course.

A speed distance Problem:
Imagine you are traveling in vehicle on a straight road and you can read the speedometer and a clock, but the odometer is broken. Further imagine that there are no road signs to tell you how far you have traveled.

You might figure out the distance you are traveling as follows.
Pick a time interval, call it Δt, maybe 60 seconds.
During the interval write down the minimum speed of the car v_i
and the maximum speed of the car V_i

Doing this you two sets of distances:

$$d_i = v_i \Delta t \le V_i \Delta t = D_i$$

Summing these distances $D = vt$ you get upper and lower limits on your total distance:

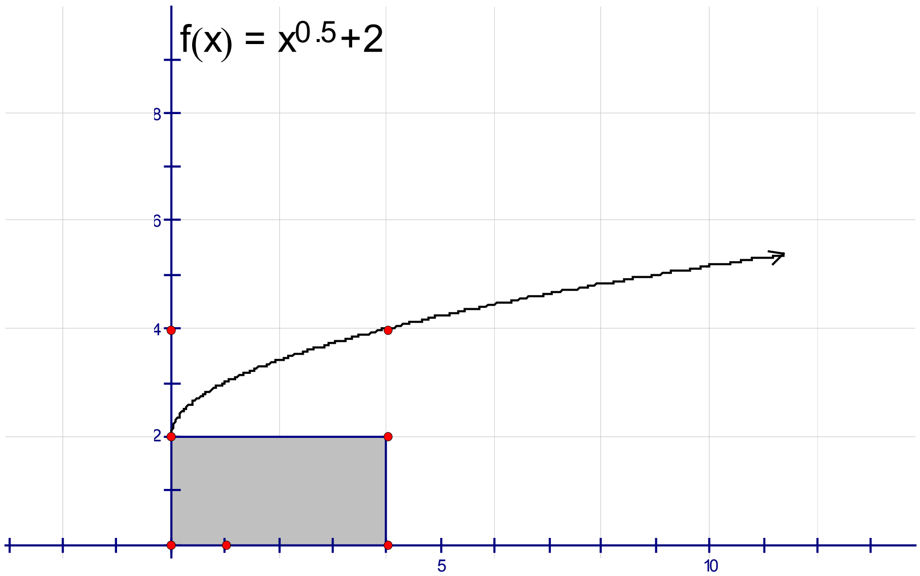
$$\sum_{i=1}^{n} d_i \le X \le \sum_{i=1}^{n} D_i$$

Now if you measure in smaller and smaller time intervals n grows larger and larger.
Taking the limit as $n \longrightarrow \infty$ both the lower and upper bounds will converge to the actual distance.

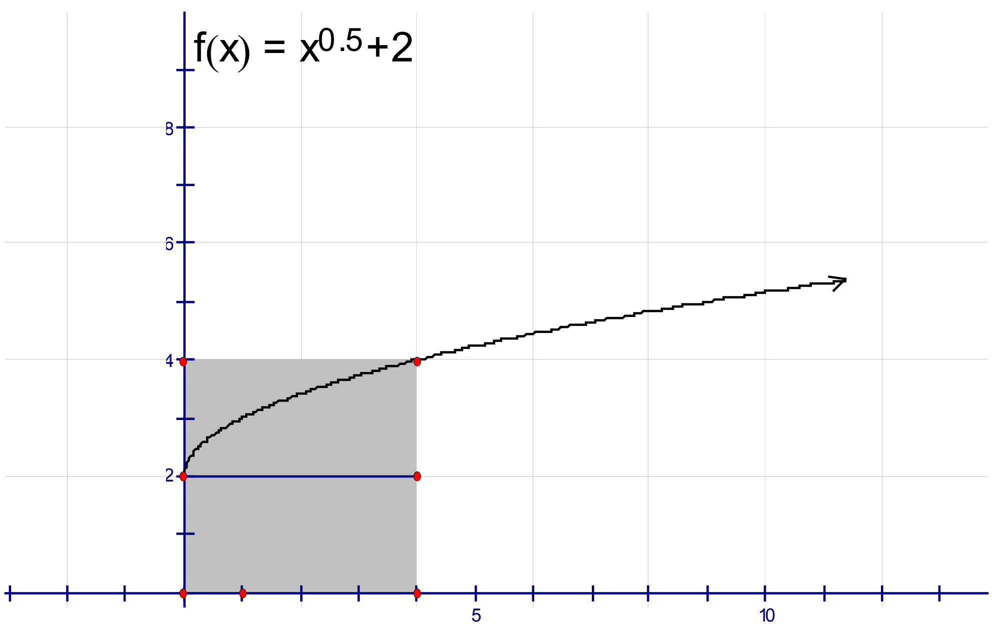
Note this is exactly the same mathematical operation we performed when doing area.

An example of Approximate Integration

As an example, we will look at finding the area under the curve $y = \sqrt{x} + 2$ between 0 and 4.

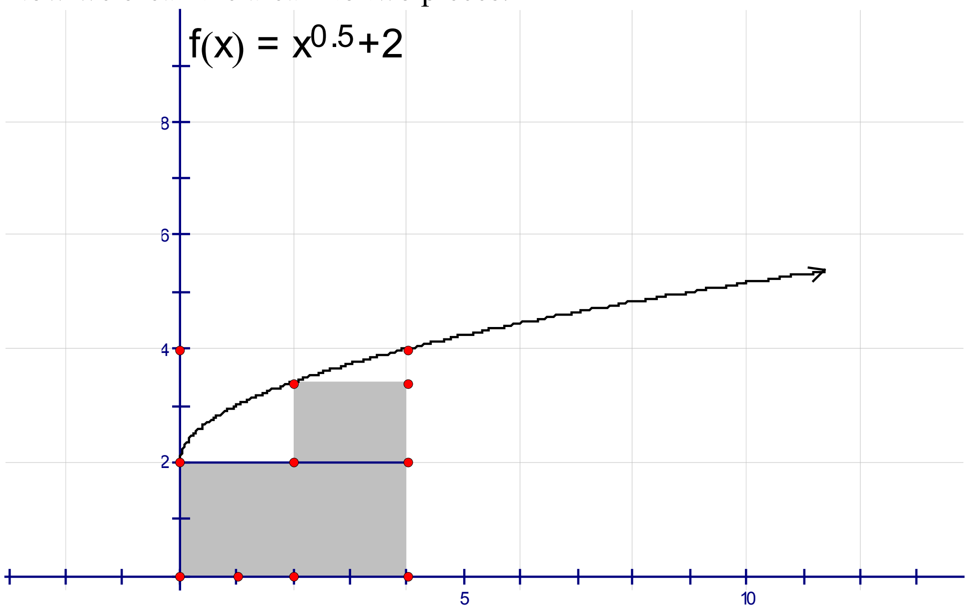


In this diagram we have estimated the area by a rectangle that is clearly less than the total area. Our lower estimate is 2 x 4 = 8

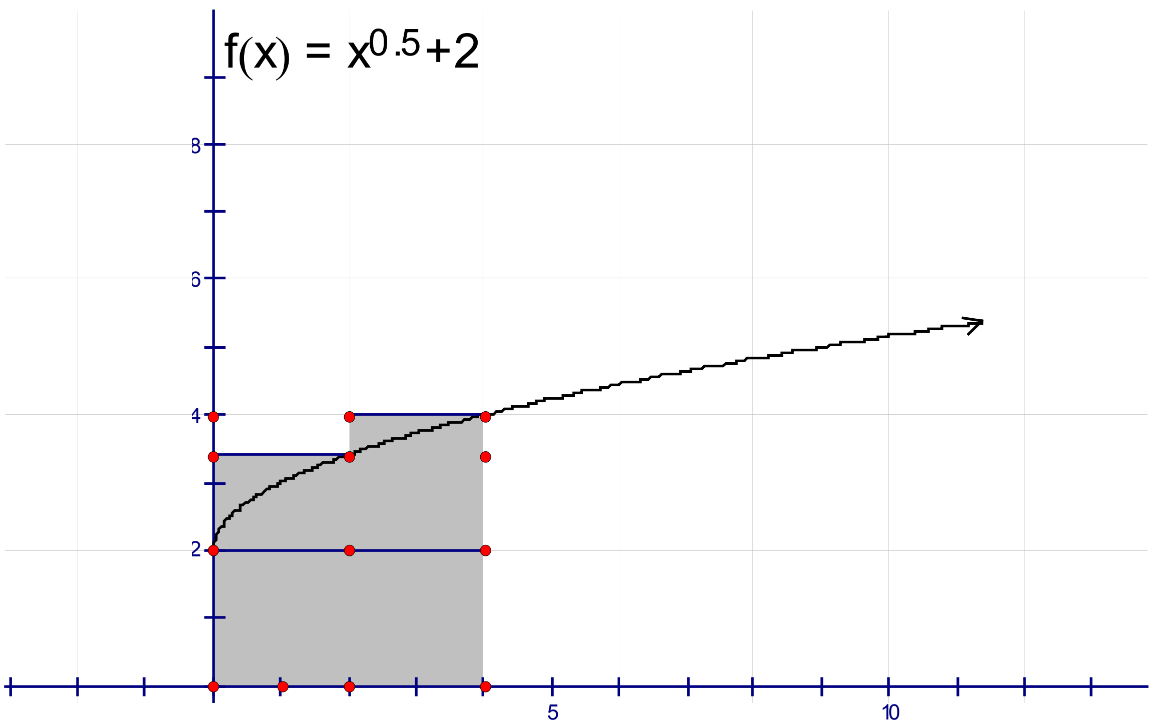


In this diagram we have estimated the area by a rectangle that is clearly greater than the total area. Our upper estimate is 4 x 4 = 16.

Now we break the area into two pieces:



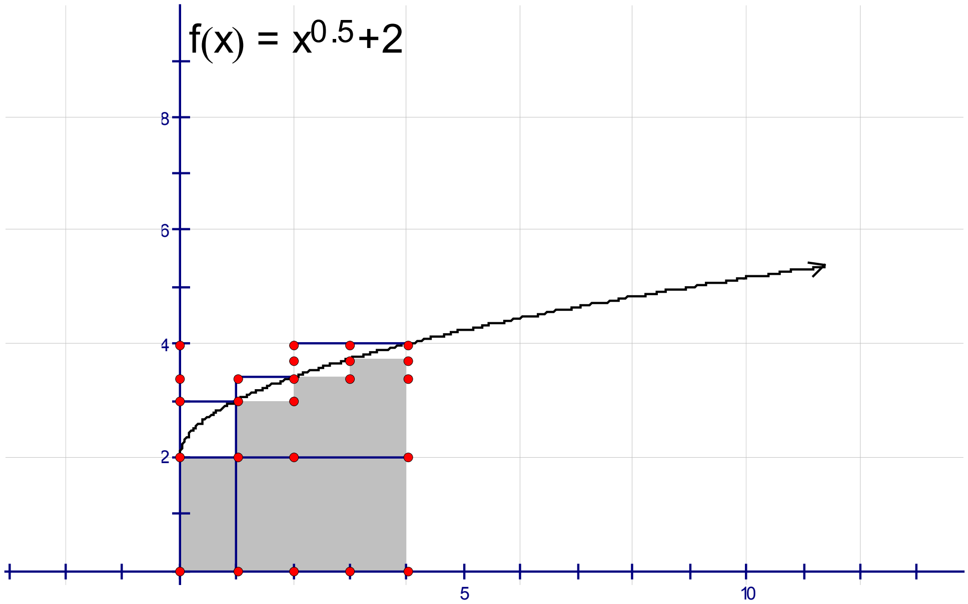
Now our lower estimate is 2 x 2 + 2 * 3.414 ≈ 10.83



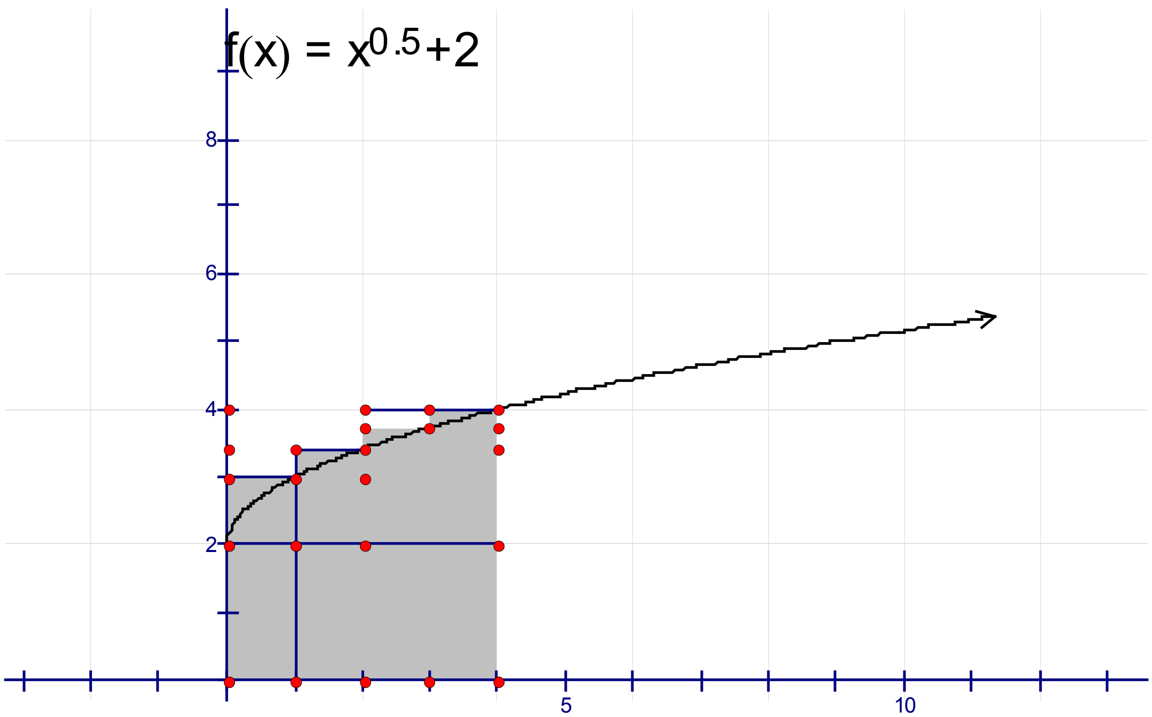
And our upper estimate is 2 x 3.414 + 2 X 4 ≈ 14.82.

The distance between our estimates has dropped from 8 to 4.
If you continue to split the width of the rectangles in two, what will happen?

One more split:



The lower estimate is now 2 x 1 + 3 x 1 + 2.73 x 1 + 3.41 x 1 ≈ 11.14



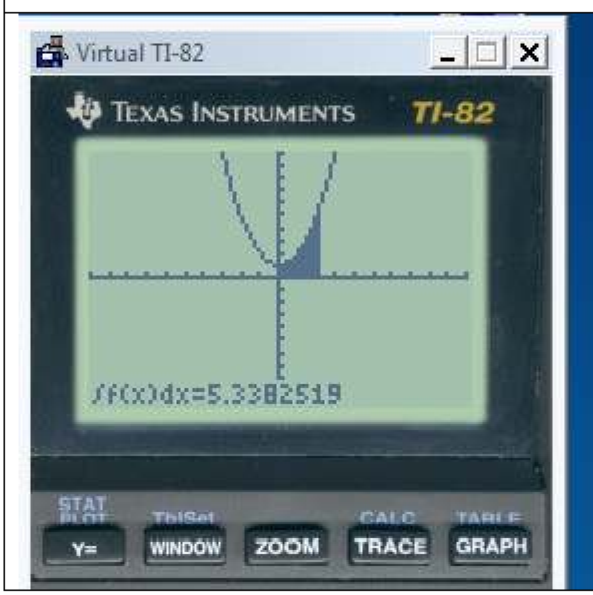
The upper estimate is 3 x 1 + 3.41 x 1 + 3.73 x 1 + 4 x 1 ≈ 14.14

Inserting these into a table we see:

rectangles	lower	upper
1	8	16
2	10.83	14.82
4	11.14	14.14

As the rectangles increase,
what happens to the lower estimates?
What happens to the upper estimates?
What do you suspect will happen to the estimates if we repeatedly increase the number rectangles?

Mechanical calculation of a definite Integral using a Ti-8x



Note that this mechanical method also produces an approximation.

Next class we will go on to learn how to calculate a definite integral exactly.