

## Lesson Plan 5 - Integration by Substitution 5.5

- 1) Take attendance
- 2) Return Homework (Point out problem to FT)
- 3) Questions on Homework 5,4 or anything else
- 4) Administer Quiz
- 5) Break

### Chapter 5.5 Substitution

Note that sometimes there is more than one substitution that will work

Example:

$$\int \sqrt{2x+1} dx$$

$$u = 2x+1 \quad \frac{du}{dx} = 2 \quad dx = \frac{du}{2}$$

$$\int \sqrt{2x+1} dx = \int \sqrt{u} \cdot \frac{1}{2} du = \frac{1}{3} u^{3/2} + C = \frac{1}{3} (2x+1)^{3/2} + C$$

alternatively

$$u^2 = 2x+1 \quad u = \sqrt{2x+1} \quad \frac{du}{dx} = \frac{1}{2} \frac{2}{\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}} \quad du = \frac{dx}{\sqrt{2x+1}} \quad du = \frac{1}{u} dx \quad dx = u du$$

$$\int \sqrt{2x+1} dx = \int \sqrt{u^2} \cdot u du = \int u^2 du = \frac{u^3}{3} = \frac{(2x+1)^{3/2}}{3} + C$$

Note: Not every substitution will work.

Here's another one

$$\int \frac{x}{\sqrt{1-4x^2}} dx$$

$$\text{Let } u = 1 - 4x^2 \quad \frac{du}{dx} = -8x \quad x dx = -\frac{du}{8}$$

$$\int \frac{x}{\sqrt{1-4x^2}} dx = \int \frac{-1}{8\sqrt{u}} du = -\frac{1}{8} 2u^{1/2} + C = -\frac{1}{4} \sqrt{1-4x^2} + C$$

Alternatively

$$\text{Let } x = \frac{1}{2} \sin u \quad \frac{dx}{du} = \frac{1}{2} \cos u \quad dx = \frac{1}{2} \cos u du$$

$$\int \frac{x}{\sqrt{1-4x^2}} dx = \int \frac{1}{2} \frac{\sin u \cdot \cos u}{\sqrt{1-\sin^2 u}} du = \int \frac{\sin u}{2} du = -\frac{\cos u}{2} + C$$

$$\text{But } x = \frac{1}{2} \sin u \longrightarrow x^2 = \frac{\sin^2 u}{4} = \frac{1-\cos^2 u}{4} \longrightarrow \cos u = \frac{\sqrt{1-4x^2}}{2}$$

$$\text{So } \int \frac{x}{\sqrt{1-4x^2}} dx = -\frac{\sqrt{1-4x^2}}{4} + C$$

Evaluating a definite integral using substitution

Recall the first Problem

$$\int \sqrt{2x+1} dx = \frac{1}{3}u^{3/2} + C = \frac{1}{3}(2x+1)^{3/2} + C$$

We can evaluate the integral  $\int_1^4 \sqrt{2x+1} dx$  in two ways

$$\left[ \frac{1}{3}(2x+1)^{3/2} \right]_1^4 = \frac{1}{3} [9^{3/2} - 3^{3/2}] = 9 - \sqrt{3}$$

Or noting that when  $x = 1$   $u = 3$  and when  $x = 4$   $u = 9$

$$\text{Plugging in } \frac{1}{3} [u^{3/2}]_3^9 = \frac{1}{3} [27 - 3\sqrt{3}] = 9 - \sqrt{3}$$

One more example:

$$\int \frac{\ln x}{x} dx$$

$$\text{Let } u = \ln x \rightarrow du = \frac{dx}{x}$$

$$\int \frac{\ln x}{x} dx = \int u du = \frac{u^2}{2} = \frac{\ln^2 x}{2}$$

Homework Assignment:

5.5 p. 381 7, 8, 13, 19, 24, 32, 44, 63, 67, 68 (Don't sweat 67-68 if you don't understand them.)