6.1 EXERCISES

CONCEPTS

1. (a) The radian measure of an angle θ is the length of the _____ that subtends the angle in a circle of radius _____.

   (b) To convert degrees to radians, we multiply by _____.

   (c) To convert radians to degrees, we multiply by _____.

2. A central angle θ is drawn in a circle of radius r.

   (a) The length of the arc subtended by θ is $s =$ _____.

   (b) The area of the sector with central angle θ is $A =$ _____.

SKILLS

3–14 ■ Find the radian measure of the angle with the given degree measure.

3. $72°$
4. $54°$
5. $-45°$
6. $-60°$
7. $-75°$
8. $-300°$
9. $1080°$
10. $3960°$
11. $96°$
12. $15°$
13. $7.5°$
14. $202.5°$

15–26 ■ Find the degree measure of the angle with the given radian measure.

15. $\frac{7\pi}{6}$
16. $\frac{11\pi}{3}$
17. $-\frac{5\pi}{4}$
18. $-\frac{3\pi}{2}$
19. 3
20. -2
21. -1.2
22. 3.4
23. $\frac{\pi}{10}$
24. $\frac{5\pi}{18}$
25. $-\frac{2\pi}{15}$
26. $-\frac{13\pi}{12}$

27–32 ■ The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal with the given angle.

27. $50°$
28. $135°$
29. $\frac{3\pi}{4}$
30. $\frac{11\pi}{6}$
31. $-\frac{\pi}{4}$
32. $-45°$

33–38 ■ The measures of two angles in standard position are given. Determine whether the angles are coterminal.

33. $70°$, $430°$
34. $-30°$, $330°$
35. $\frac{5\pi}{6}$, $\frac{17\pi}{6}$
36. $\frac{32\pi}{3}$, $\frac{11\pi}{3}$
37. $155°$, $875°$
38. $50°$, $340°$

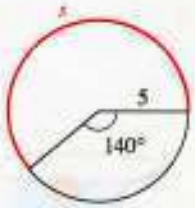
39–44 ■ Find an angle between $0°$ and $360°$ that is coterminal with the given angle.

39. $733°$
40. $361°$
41. $1110°$
42. $-100°$
43. $-800°$
44. $1270°$

45–50 ■ Find an angle between 0 and 2π that is coterminal with the given angle.

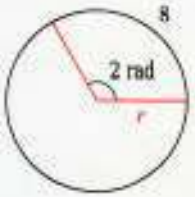
45. $\frac{17\pi}{6}$
46. $-\frac{7\pi}{3}$
47. 87π
48. 10
49. $\frac{17\pi}{4}$
50. $\frac{51\pi}{2}$

51. Find the length of the arc s in the figure.



52. Find the angle θ in the figure.

53. Find the radius r of the circle in the figure.



54. Find the length of an arc that subtends a central angle of $45°$ in a circle of radius 10 m.

55. Find the length of an arc that subtends a central angle of 2 rad in a circle of radius 2 mi.

56. A central angle θ in a circle of radius 5 m is subtended by an arc of length 6 m. Find the measure of θ in degrees and in radians.

57. An arc of length 100 m subtends a central angle θ in a circle of radius 50 m. Find the measure of θ in degrees and in radians.

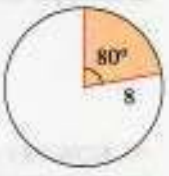
58. A circular arc of length 3 ft subtends a central angle of $25°$. Find the radius of the circle.

59. Find the radius of the circle if an arc of length 6 m on the circle subtends a central angle of $\pi/6$ rad.

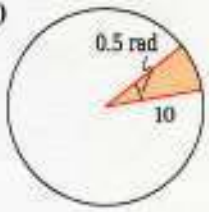
60. Find the radius of the circle if an arc of length 4 ft on the circle subtends a central angle of $135°$.

61. Find the area of the sector shown in each figure.

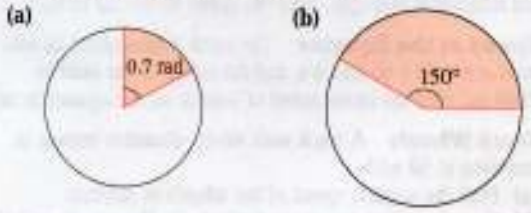
(a)



(b)



62. Find the radius of each circle if the area of the sector is 12.



63. Find the area of a sector with central angle 1 rad in a circle of radius 10 m.

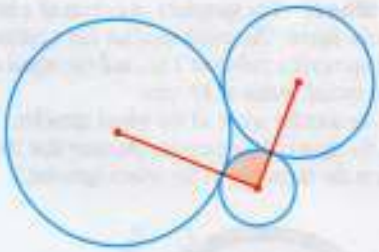
64. A sector of a circle has a central angle of 60°. Find the area of the sector if the radius of the circle is 3 mi.

65. The area of a sector of a circle with a central angle of 2 rad is 16 m^2. Find the radius of the circle.

66. A sector of a circle of radius 24 mi has an area of 288 mi^2. Find the central angle of the sector.

67 The area of a circle is 72 cm^2. Find the area of a sector of this circle that subtends a central angle of $\pi/6$ rad.

68. Three circles with radii 1, 2, and 3 ft are externally tangent to one another, as shown in the figure. Find the area of the sector of the circle of radius 1 that is cut off by the line segments joining the center of that circle to the centers of the other two circles.

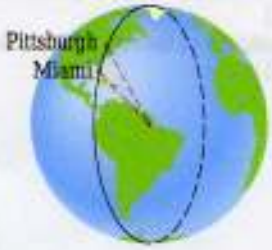


APPLICATIONS

69. Travel Distance A car's wheels are 28 in. in diameter. How far (in miles) will the car travel if its wheels revolve 10,000 times without slipping?

70. Wheel Revolutions How many revolutions will a car wheel of diameter 30 in. make as the car travels a distance of one mile?

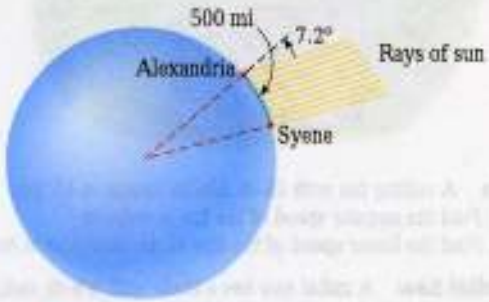
71. Latitudes Pittsburgh, Pennsylvania, and Miami, Florida, lie approximately on the same meridian. Pittsburgh has a latitude of 40.5° N, and Miami has a latitude of 25.5° N. Find the distance between these two cities. (The radius of the earth is 3960 mi.)



72. Latitudes Memphis, Tennessee, and New Orleans, Louisiana, lie approximately on the same meridian. Memphis has a latitude of 35° N, and New Orleans has a latitude of 30° N. Find the distance between these two cities. (The radius of the earth is 3960 mi.)

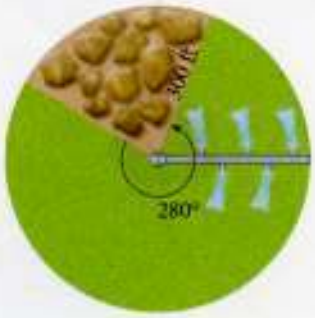
73. Orbit of the Earth Find the distance that the earth travels in one day in its path around the sun. Assume that a year has 365 days and that the path of the earth around the sun is a circle of radius 93 million miles. [The path of the earth around the sun is actually an *ellipse* with the sun at one focus (see Section 10.2). This ellipse, however, has very small eccentricity, so it is nearly circular.]

74. Circumference of the Earth The Greek mathematician Eratosthenes (ca. 276–195 B.C.) measured the circumference of the earth from the following observations. He noticed that on a certain day the sun shone directly down a deep well in Syene (modern Aswan). At the same time in Alexandria, 500 miles north (on the same meridian), the rays of the sun shone at an angle of 7.2° to the zenith. Use this information and the figure to find the radius and circumference of the earth.

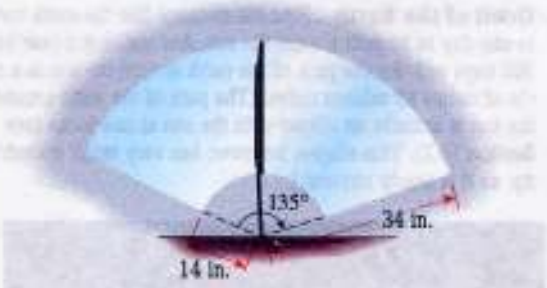


75. Nautical Miles Find the distance along an arc on the surface of the earth that subtends a central angle of 1 minute (1 minute = $\frac{1}{60}$ degree). This distance is called a *nautical mile*. (The radius of the earth is 3960 mi.)

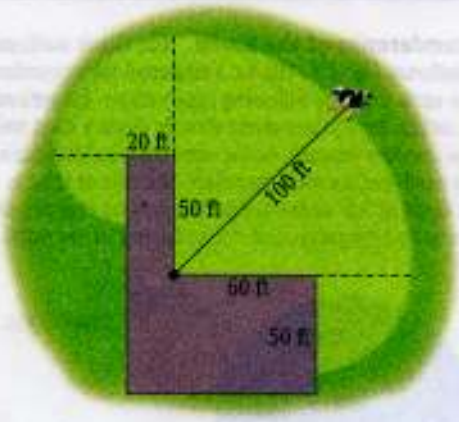
76. Irrigation An irrigation system uses a straight sprinkler pipe 300 ft long that pivots around a central point as shown. Due to an obstacle the pipe is allowed to pivot through 280° only. Find the area irrigated by this system.



77. **Windshield Wipers** The top and bottom ends of a windshield wiper blade are 34 in. and 14 in., respectively, from the pivot point. While in operation, the wiper sweeps through 135°. Find the area swept by the blade.



78. **The Tethered Cow** A cow is tethered by a 100-ft rope to the inside corner of an L-shaped building, as shown in the figure. Find the area that the cow can graze.



79. **Fan** A ceiling fan with 16-in. blades rotates at 45 rpm.
(a) Find the angular speed of the fan in rad/min.
(b) Find the linear speed of the tips of the blades in in./min.

80. **Radial Saw** A radial saw has a blade with a 6-in. radius. Suppose that the blade spins at 1000 rpm.
(a) Find the angular speed of the blade in rad/min.
(b) Find the linear speed of the sawteeth in ft/s.

81. **Winch** A winch of radius 2 ft is used to lift heavy loads. If the winch makes 8 revolutions every 15 s, find the speed at which the load is rising.

82. **Speed of a Car** The wheels of a car have radius 11 in. and are rotating at 600 rpm. Find the speed of the car in mi/h.

83. **Speed at the Equator** The earth rotates about its axis once every 23 h 56 min 4 s, and the radius of the earth is 3960 mi. Find the linear speed of a point on the equator in mi/h.

84. **Truck Wheels** A truck with 48-in.-diameter wheels is traveling at 50 mi/h.
(a) Find the angular speed of the wheels in rad/min.
(b) How many revolutions per minute do the wheels make?

85 **Speed of a Current** To measure the speed of a current, scientists place a paddle wheel in the stream and observe the rate at which it rotates. If the paddle wheel has radius 0.20 m and rotates at 100 rpm, find the speed of the current in m/s.

86. **Bicycle Wheel** The sprockets and chain of a bicycle are shown in the figure. The pedal sprocket has a radius of 4 in., the wheel sprocket a radius of 2 in., and the wheel a radius of 13 in. The cyclist pedals at 40 rpm.
(a) Find the angular speed of the wheel sprocket.
(b) Find the speed of the bicycle. (Assume that the wheel turns at the same rate as the wheel sprocket.)

87 **Conical Cup** A conical cup is made from a circular piece of paper with radius 6 cm by cutting out a sector and joining the edges as shown on the next page. Suppose $\theta = 5\pi/3$.
(a) Find the circumference C of the opening of the cup.
(b) Find the radius r of the opening of the cup. [*Hint:* Use $C = 2\pi r$.]
(c) Find the height h of the cup. [*Hint:* Use the Pythagorean Theorem.]