Lesson Plan 2 - Periodic Functions

1) Attendance

2) Homework

3) Periodic Functions

In Mathematics

Formal Definition

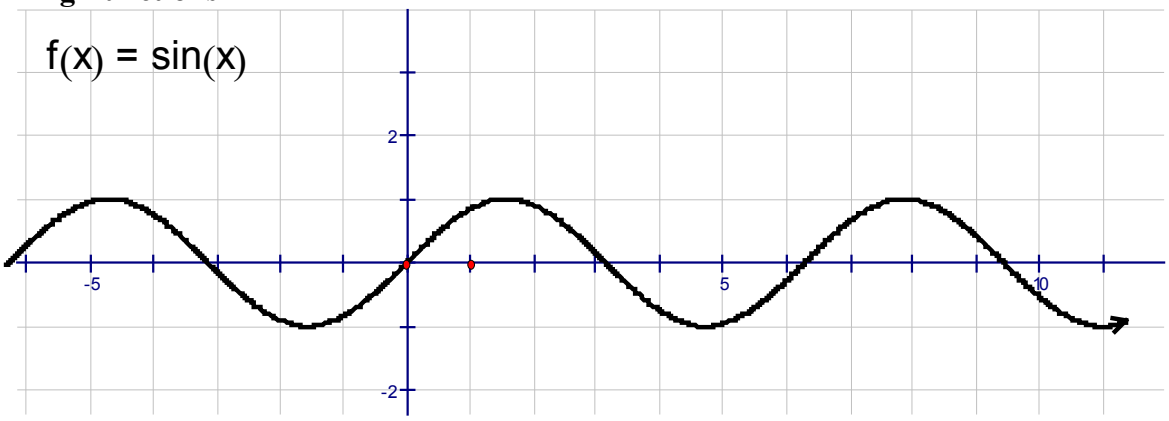
A function f is period if $f(x+p)=f(x)$ for all x in the domain of f where p is a positive constant called the **period** of the function.

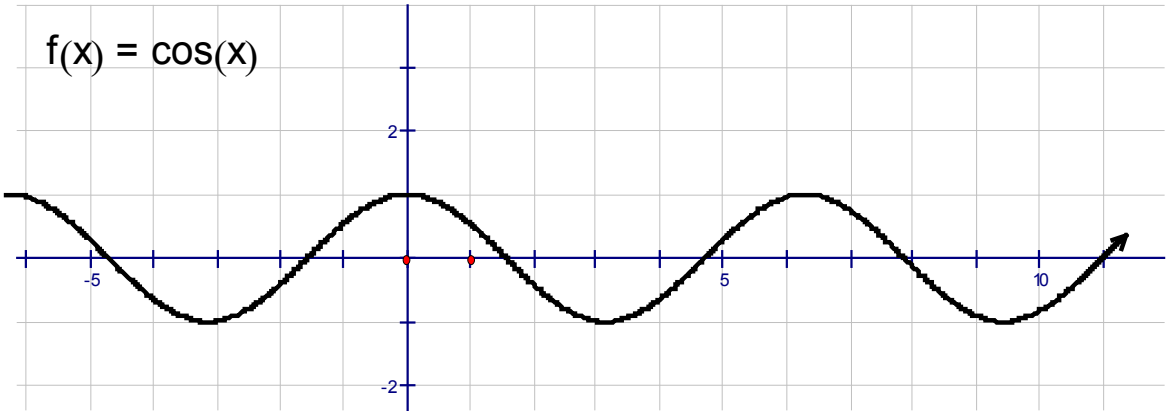
Note that if a function has period p it also will have period $2p$, $3p$, etc.
In general we will be interested in the smallest or shortest period.

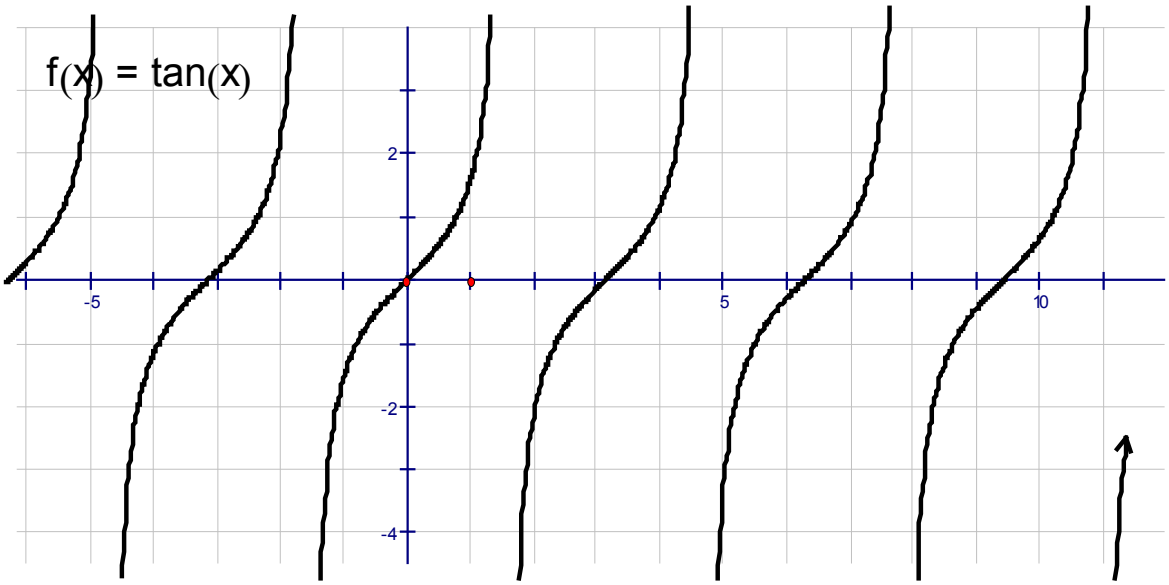
One way to think about a periodic function, is that it is a function that you can do a transformation on the function along the x axis and end up with the same function.

Examples of periodic functions:

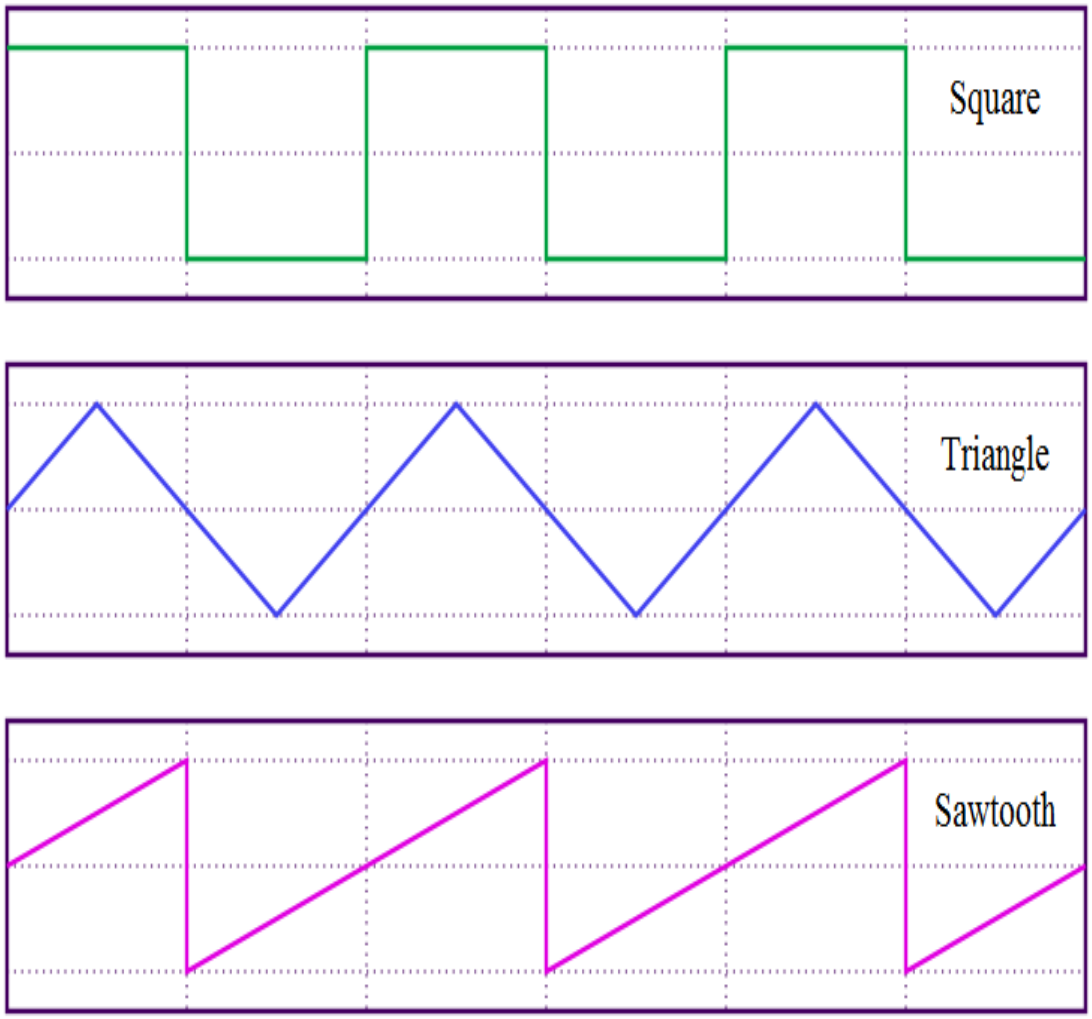
Trig Functions





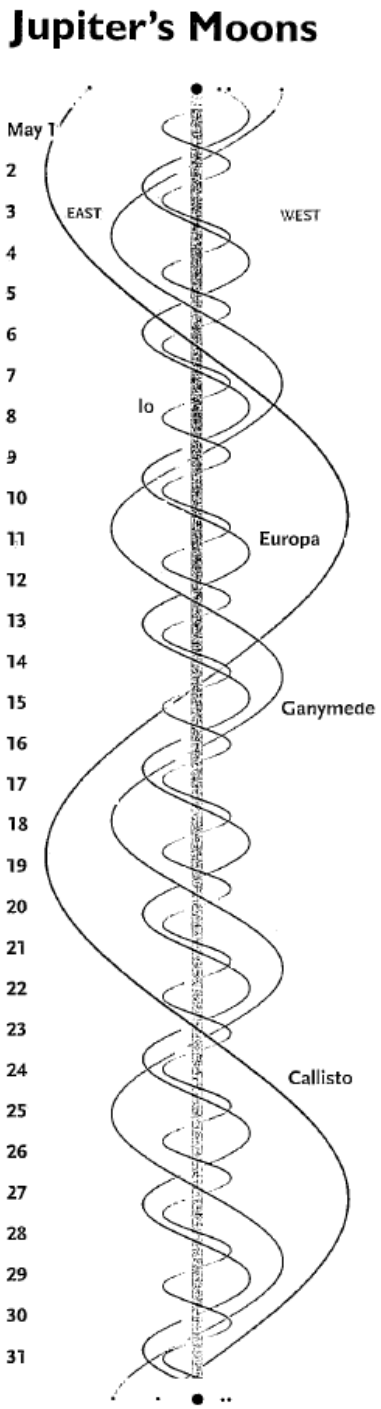


Square, Triangle and Sawtooth waves



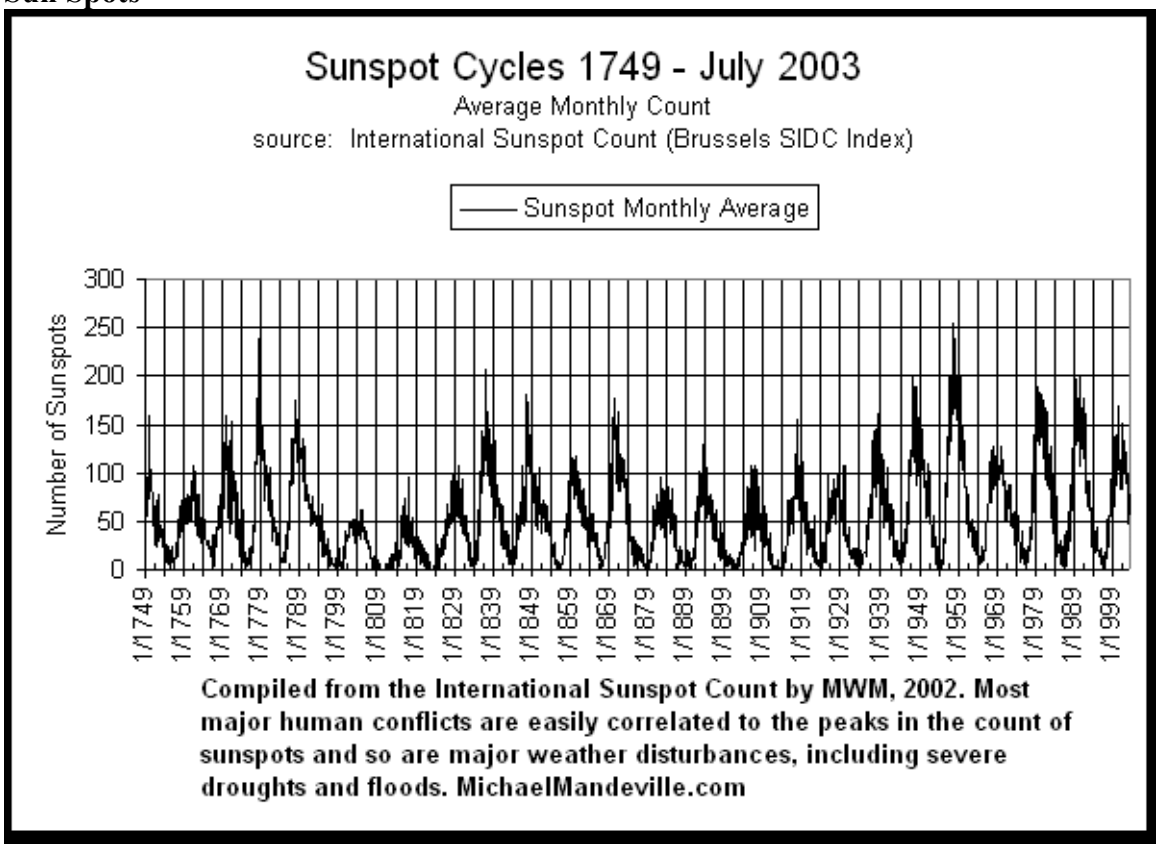
Examples of phenomenon from the real world that are modeled by periodic functions

Orbits of planets and moons:



The wavy lines represent Jupiter's four big satellites. The central vertical band is Jupiter itself. Each gray or black horizontal band is one day, from 0^h (upper edge of band) to 24^h UT (GMT). UT dates are at left. Slide a paper's edge down to your date and time, and read across to see the satellites' positions east or west of Jupiter.

Sun Spots



Tides

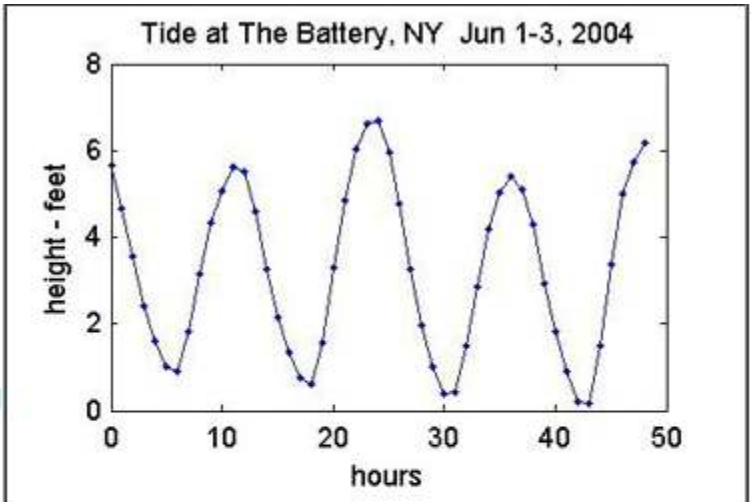
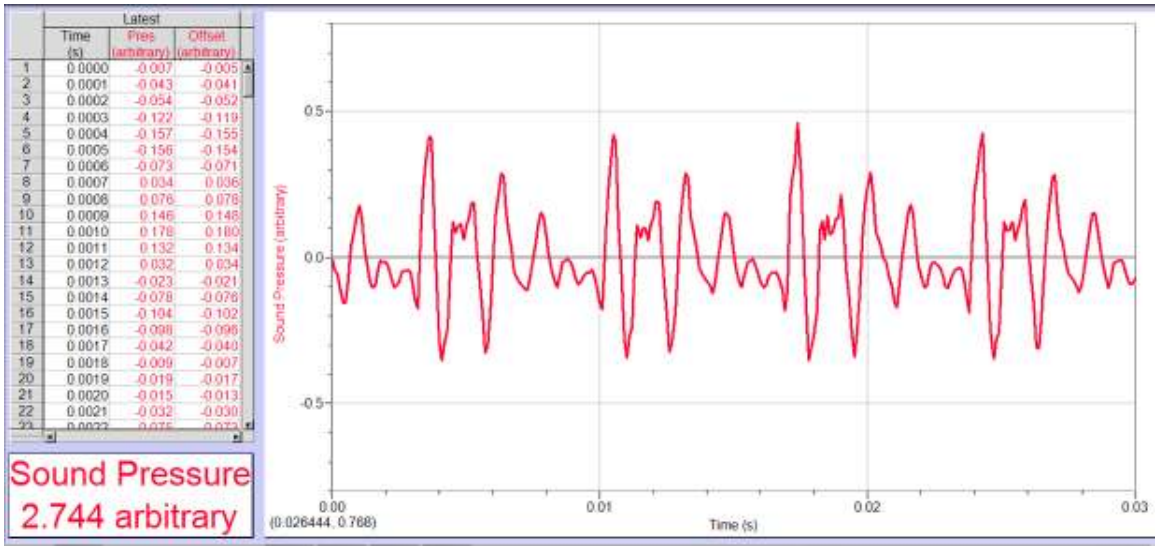


Figure 5 Height of tide as measured at The Battery - the southern end of Manhattan. The measurements were taken at one hour intervals. The graphs shows the normal semidiurnal tide pattern, with one tide cycle taking about 12 hours. *(image courtesy of NOAA)*

Sound Waves



http://www.schoenbrun.com/foothill/math48c-2/mpeg/SoundWaves-2.00.mp4

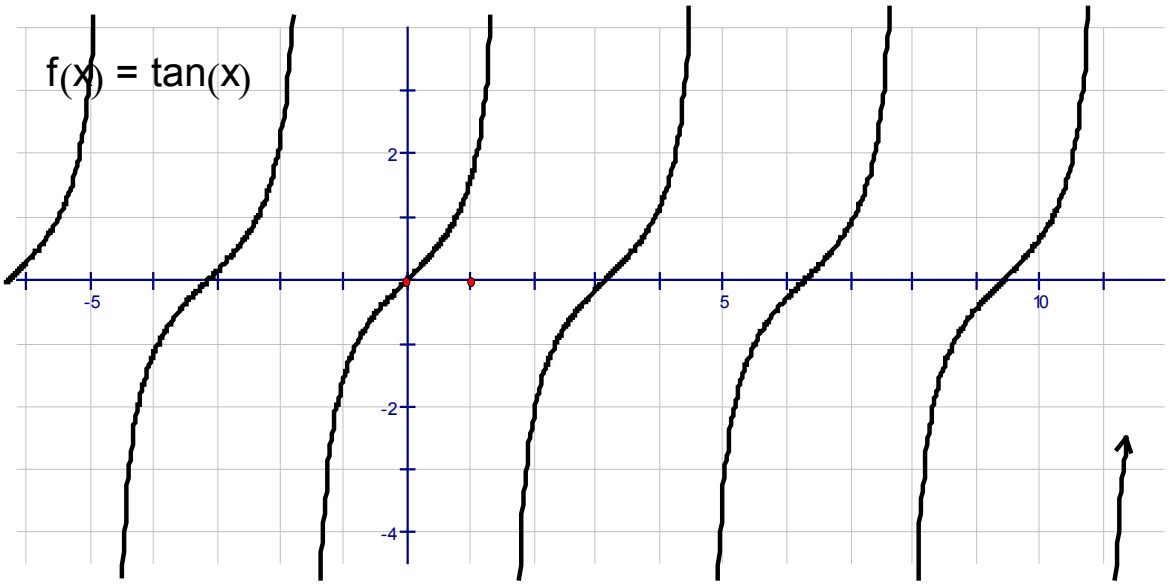
Important features of periodic functions

The Midline

One definition of a periodic function is defined as

$$y = \frac{\max \text{ of } f + \min \text{ of } f}{2}$$

You will find this in the book on page 495, however it is incomplete.
The function $f = \tan(x)$ has a midline at y=0, however you can see it has no max or min.



The Amplitude

The amplitude of a periodic function is defined as

$$y = \frac{\max \text{ of } f - \min \text{ of } f}{2}$$

Note that for our example of *tan*(*x*) the amplitude is infinite or does not exist

Frequency

For a time based function, the period has the units of time. In this case we sometimes call the period a **cycle**.

The period then can be described as cycles/time or for example cycles per second.

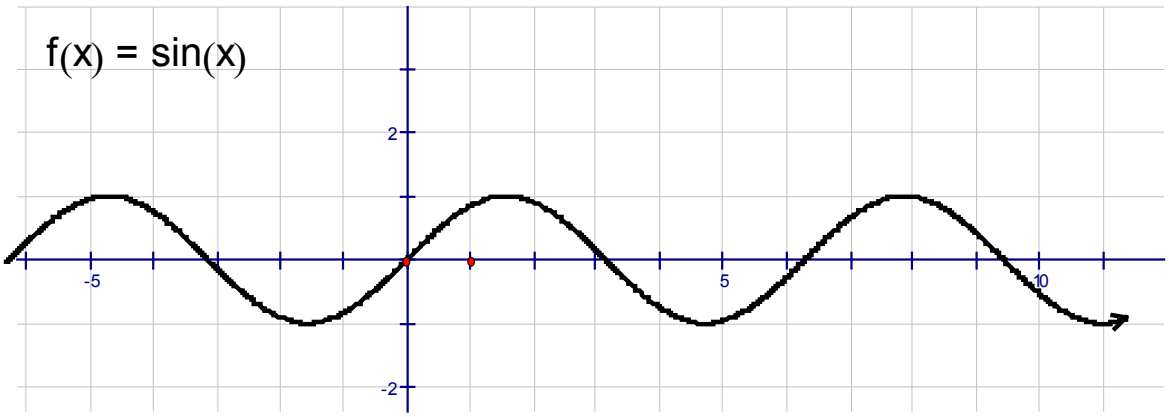
The reciprocal of a time based function is called the **frequency** and has the units cycles/time, for example 1 cycle per second.

We will use the term frequency whether for a periodic function whether it is time based or not.

Range

Just like any other kind of function, the range of a periodic function is the set of all values a function can have.

Question: What is the period, frequency, midline, amplitude and range of this sine function?



Modeling periodic functions

Given a set of tabular data that we believe is periodic, we can extract some details:

Month	Hours of daylight
Jan	9.37
Feb	10.73
Mar	12.2
Apr	13.78
May	15.05
Jun	15.62
Jul	15.1
Aug	13.8
Sep	12.23
Oct	10.7
Nov	9.37
Dec	8.77
Jan	9.35
Feb	10.73
Mar	12.23
Apr	13.82
May	15.08
Jun	15.62
Jul	
Aug	
Sep	
Oct	
Nov	
Dec	
Jan	

Find the max and min

Min = Dec 8.77
Max = June 15.62

So the midline = $\frac{8.77+15.62}{2} = 12.2$

The Amplitude = $\frac{15.62-8.77}{2} = 3.43$

From peak to peak we have June -> June = 12 months
so the period is 12 months.
The frequency is 1/12 cycles per month

Since the data seems periodic we can estimate the missing values

Homework for Monday
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