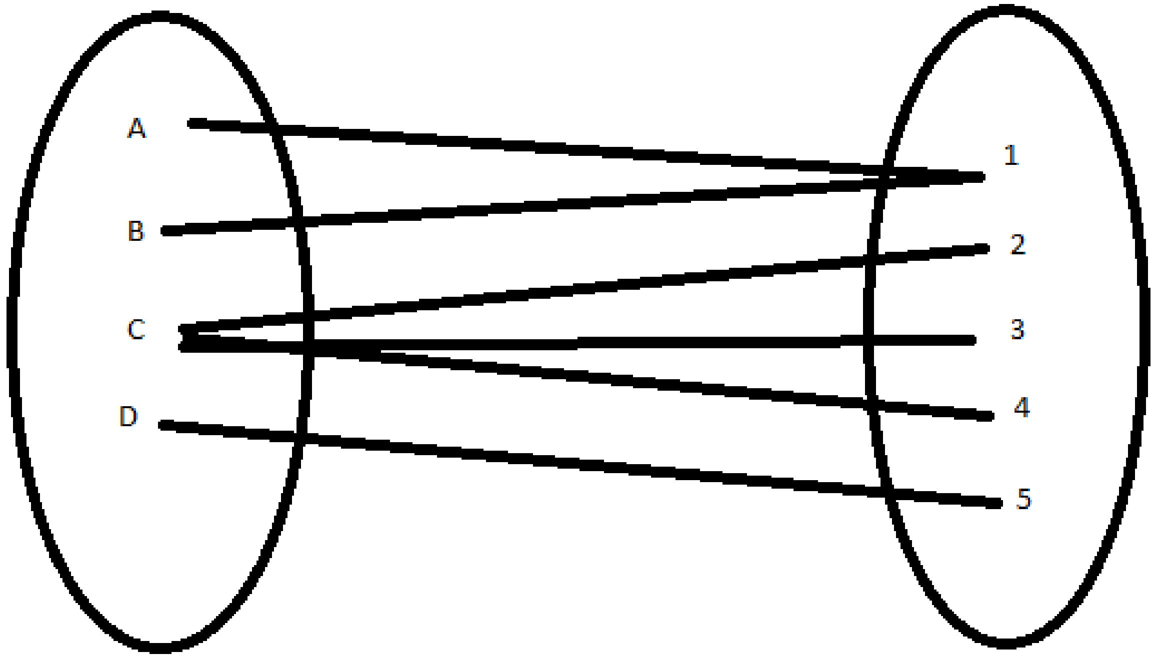
Lesson Plan 7 Inverse Trig Functions 8.7 Math 48C Mitchell Schoenbrun

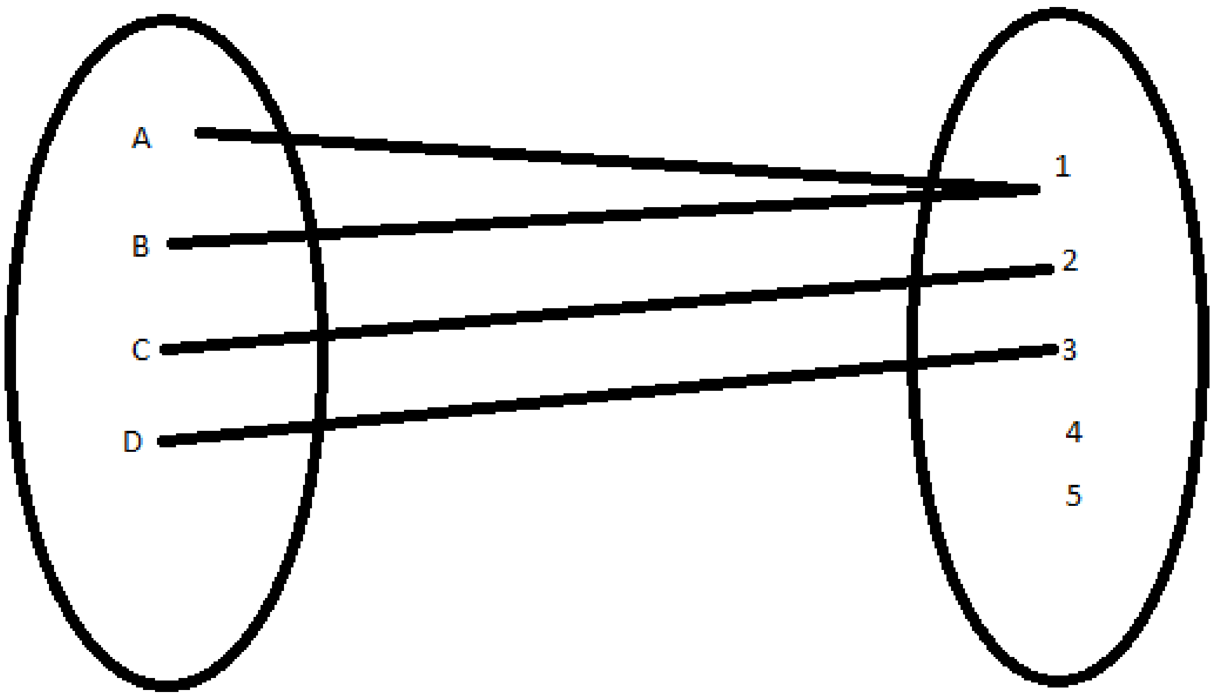
1) Attendance
2) Previous Homework problem 42, How to use calculator
3) Questions about homework 6?

Define a Relation and a Function

A relation is a mapping between two sets

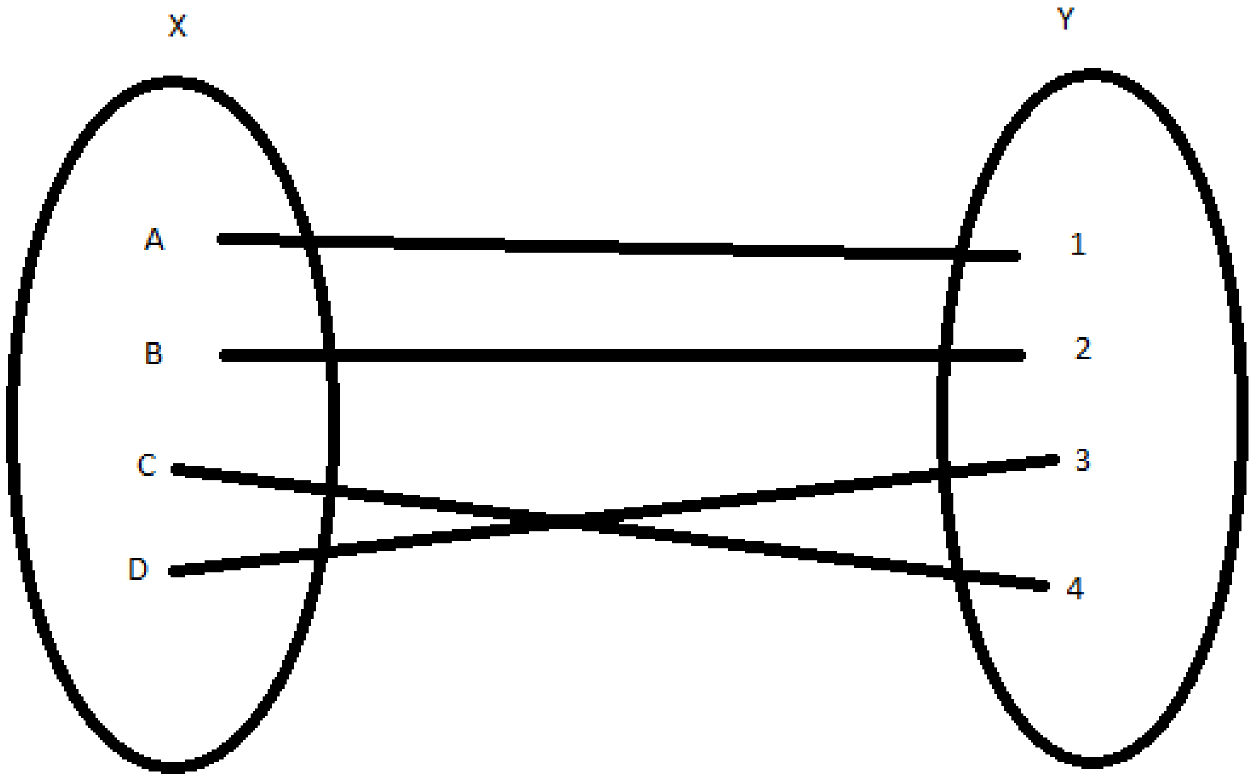


Note that this relationship is NOT a function.   Why?

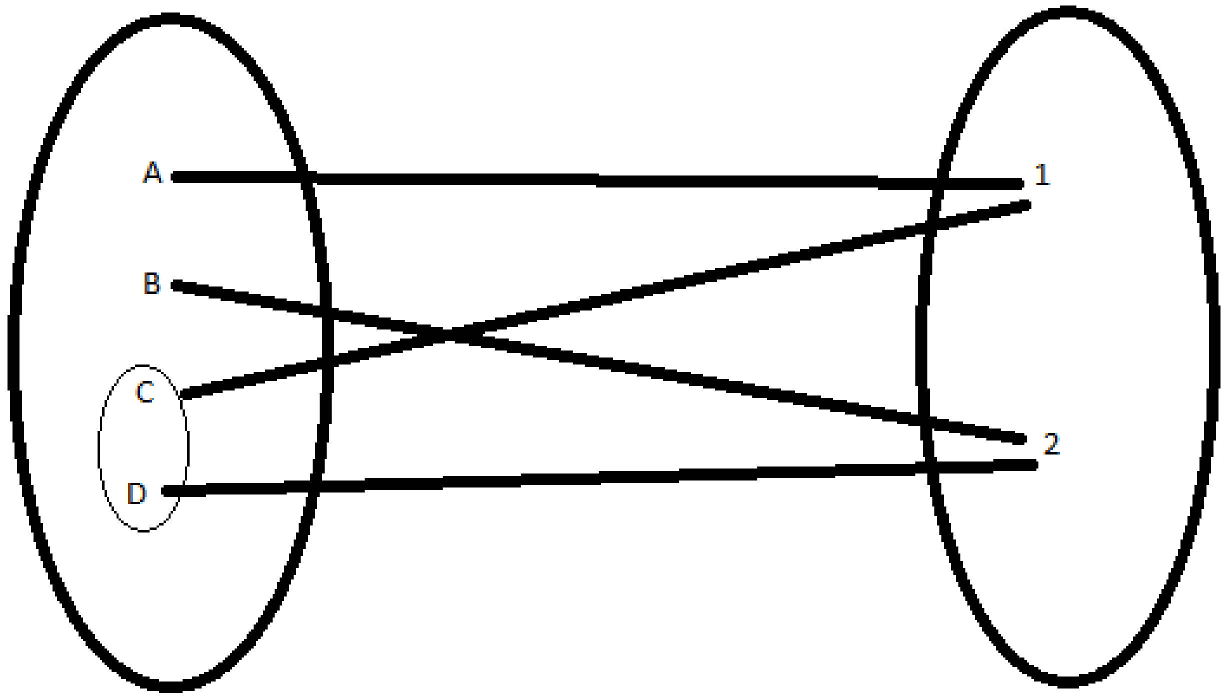


This relation is a function.   Why?

This is a function from X ---> Y with an inverse from Y----> X

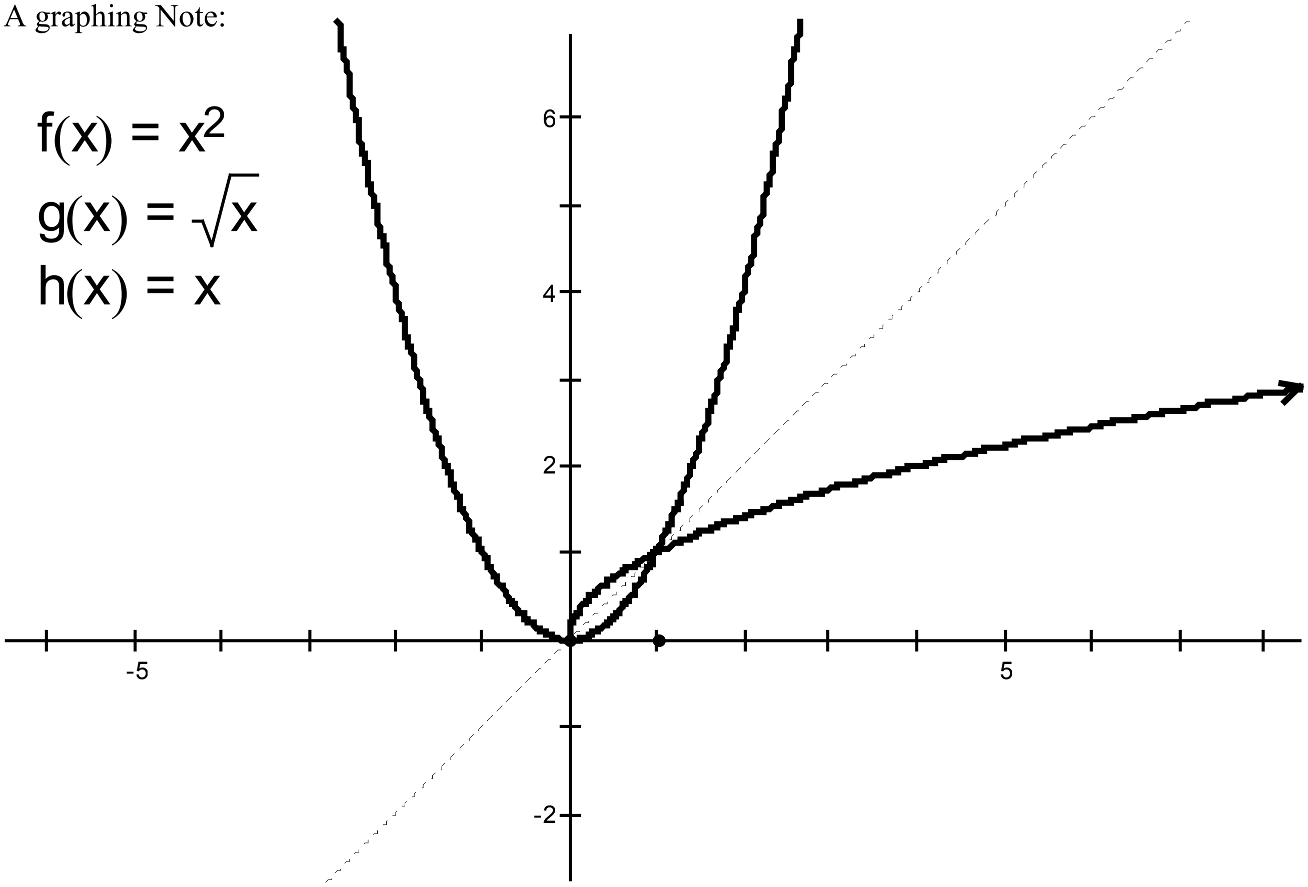


Sometimes we can give a function an inverse by truncating the Domain



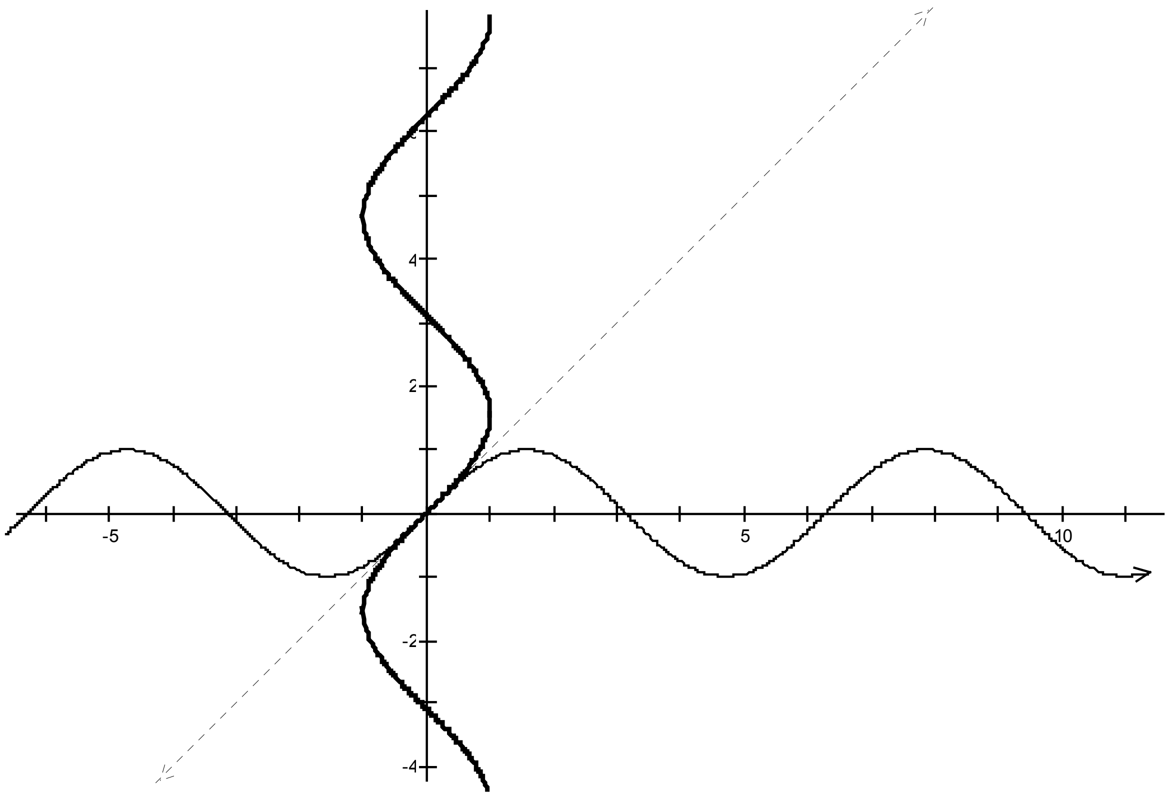
Note that by removing C and D from the domain, we now have a function that has an inverse.

A graphing Note:



You can graph the inverse function by rotating along the line y=x.

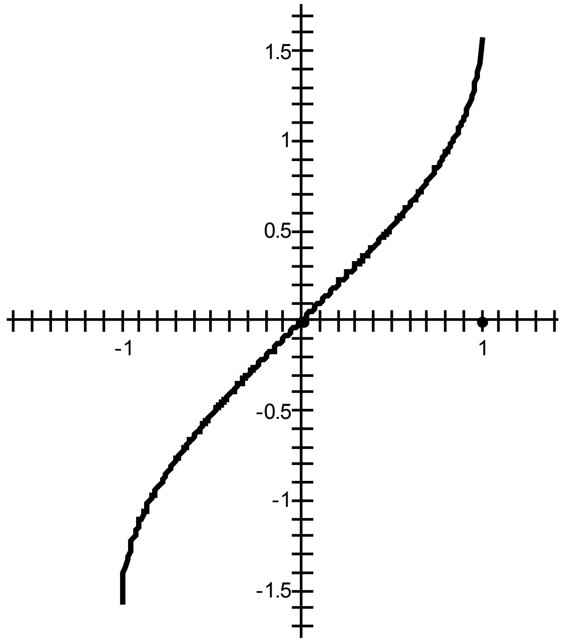
How did we need to truncate the domain to make the inverse a function?



If you do this with the sine function, the result is not a function anymore.

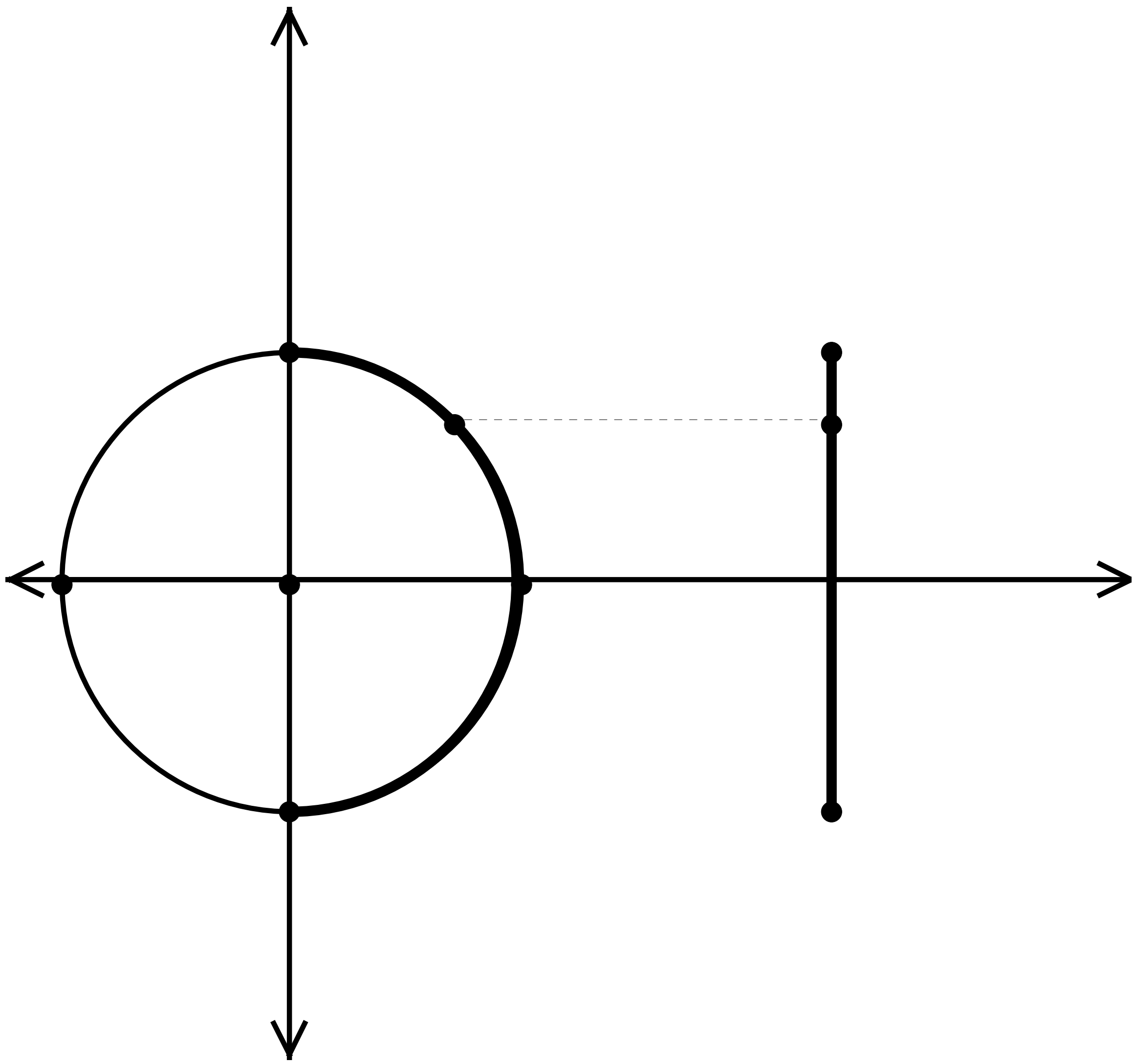
So we must truncate the Domain of Sine so that it's inverse exists.

We truncate the Domain to be $[-\pi, \pi]$



This is arbitrary, however it is the standard way of doing this.
You will note that this is what your calculator does.

Back to The Sine Function and it's inverse.



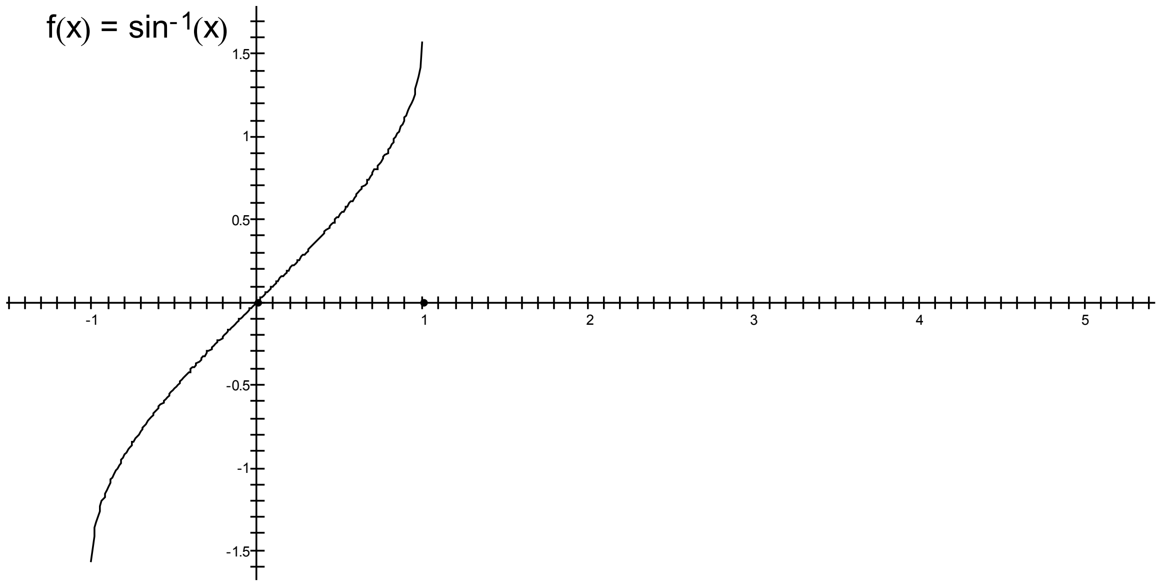
We limit the Sine function to $[-\pi, \pi]$.

The inverse of the Sine function is written either

$\arcsin(x)$

or

$\sin^{-1}(x)$

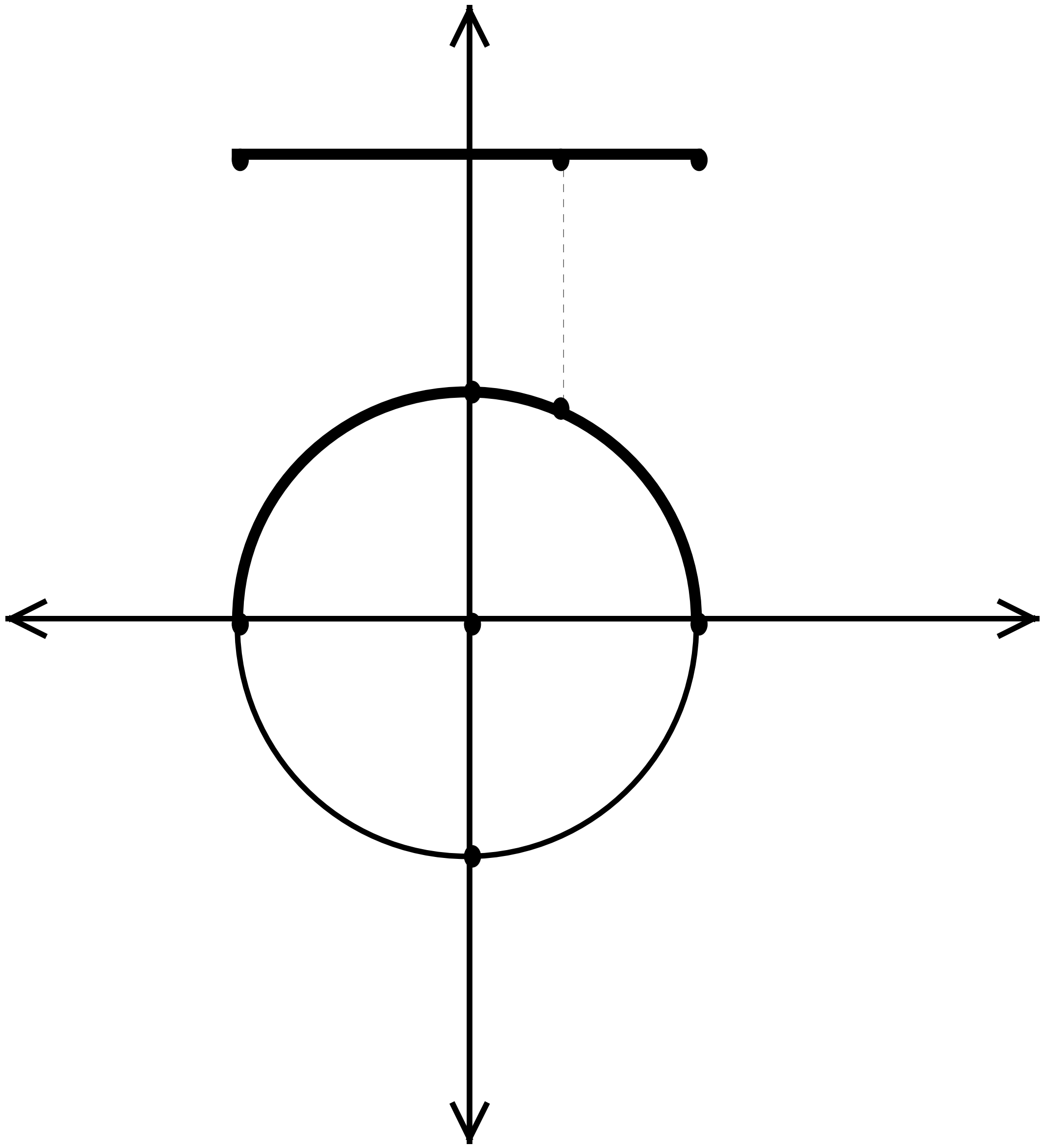


The domain of this function [-1, 1] is the range of it's inverse.
The range of this function $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ is the domain of it's inverse.

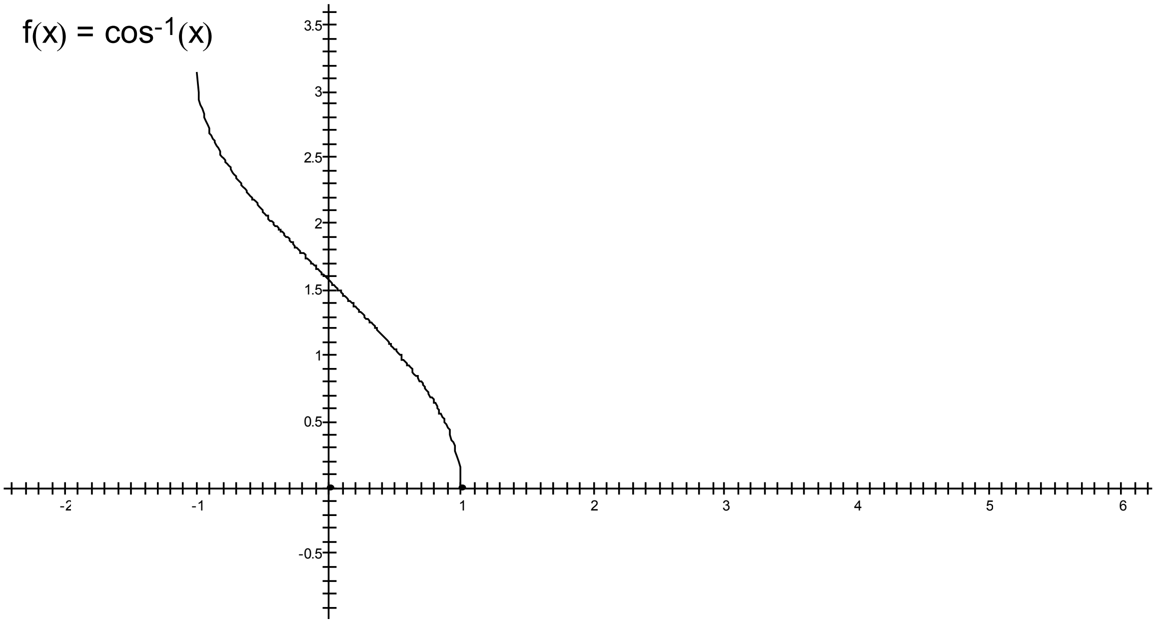
You can use your calculator to find specific values of an inverse function:

Note that the resulting value will depend on the MODE of the calculator
Degrees or Radians.

The Cosine's domain must be truncated a little differently to get an inverse.



Domain $=[0,\pi]$ gives us the inverse cosine as follows:

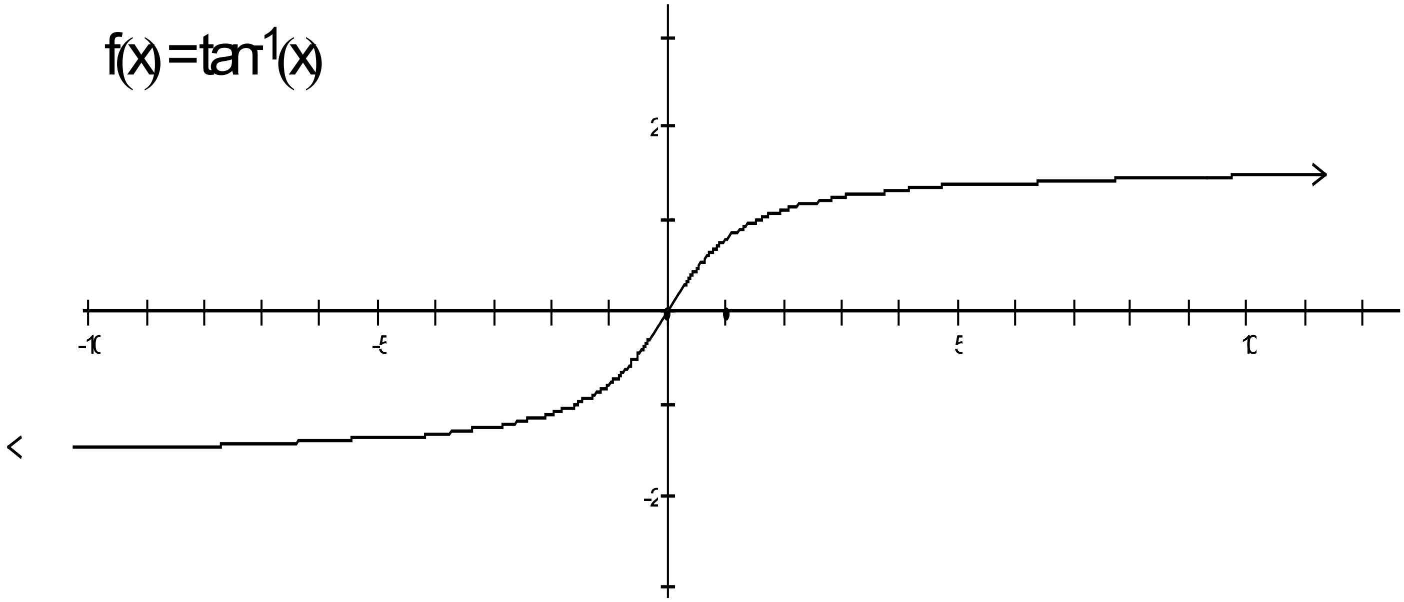


Note that it's Domain is [-1,1] like the arcsine, but it's range is $[0,\pi]$

The tangent function has a period of just π so we restrict the Domain to be $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Note that this is an open interval. Why? The end points are undefined.

The range however is $(-\infty, +\infty)$



The functions described can help you find the angle whose sine, cosine, or tangent is a specific value.

However you should be aware that this angle is not unique.

Example:

Find the angle whose sine is $\frac{1}{\sqrt{2}}$

Since $\sin\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$$

However the sine is positive in both the first and 2nd quadrants, so

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{3\pi}{4}$$

But since the sine is periodic we have multiple solutions:

$$\left\{\frac{\pi}{4}+2\pi n,\frac{3\pi}{4}+2\pi n\right\} where\, n\in\{...,-2,-1,0,1,2,...\}$$

Handout

Homework: 8.7 (P. 605) #1-8, 13, 15, 19, 20, 23, 24, 46 and 50 (NOTE: When asked, you DO NOT have to EXPLAIN!