Lesson Plan 8 Inverse Trig Functions Continued 8.7 Math 48C Mitchell Schoenbrun

1) Attendance
2) Questions about homework
3) Quiz on Monday, up to Monday's lesson

Trigonometric Functions

Alternative Notation:

$\sin^{-1}\theta$	arcsin θ
$\cos^{-1}\theta$	arccos θ
$\tan^{-1}\theta$	arctan θ

Note that some texts will make the following distinction

arcsin θ is a function (one output) with a range [-90°, 90°]

Arcsin θ is a relation with an infinite number out outputs.

Function	Domain	Range
Sine	$\mathbb{R}$	[-1,1]
Cosine	$\mathbb{R}$	[-1,1]
Tangent	$x \in \mathbb{R} : \cos(x) \neq 0$	$(-\infty, +\infty)$
Cotangent	$x \in \mathbb{R} : \sin(x) \neq 0$	$(-\infty, +\infty)$
Secant	$x \in \mathbb{R} : \cos(x) \neq 0$	$(-\infty, -1] \| [1, +\infty)$
Cosecant	$x \in \mathbb{R} : \sin(x) \neq 0$	$(-\infty, -1] \| [1, +\infty)$

Inverse Trigonometric functions

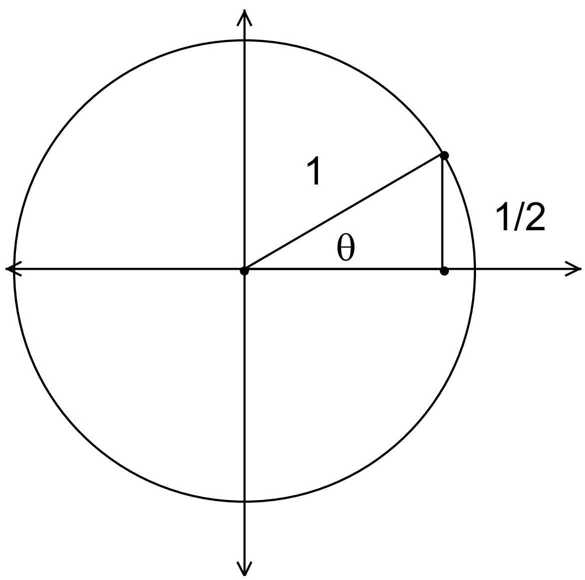
Function	Domain	Range
Inverse Sine	[-1,1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Inverse Cosine	[-1,1]	$[0, \pi]$
Inverse Tangent	$(-\infty, +\infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Inverse Cotangent	$(-\infty, +\infty)$	$[0, \pi]$
Inverse Secant	$(-\infty, -1] \| [1, +\infty)$	$[0, \pi]$
Inverse Cosecant	$(-\infty, -1] \| [1, +\infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Note the Range of a function becomes the Domain of it's inverse function.
The Range of an inverse function is smaller than the original function.

1) Solving Simple Inverse Trig problems

Exact!

Example: What is the angle whose sine is $\frac{1}{2}$, $\sin^{-1}\left(\frac{1}{2}\right) =$ ______________________



We can see immediately that this is a 30/60/90 triangle, so the angle must be 30°.

This is the principle value of $\sin^{-1}\left(\frac{1}{2}\right)$

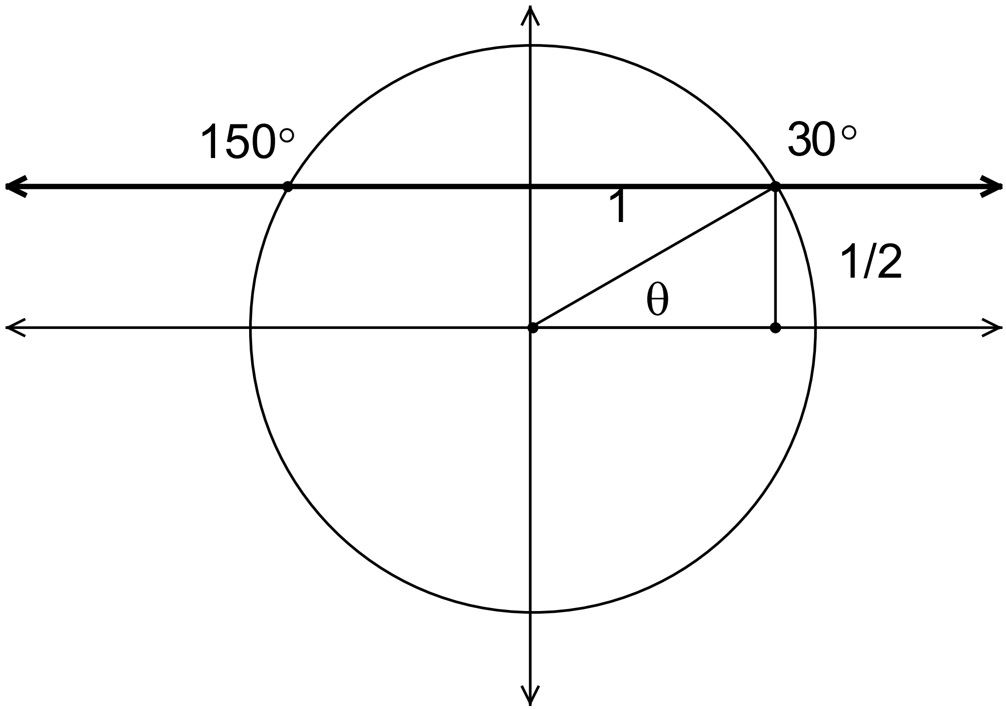
The sine is also positive in quadrant II so we also have $\sin^{-1}\left(\frac{1}{2}\right) = 180°\text{-}30°\text{=}150°$.

Finally, we can find all possible solutions by adding $2\pi n$ to each of these.

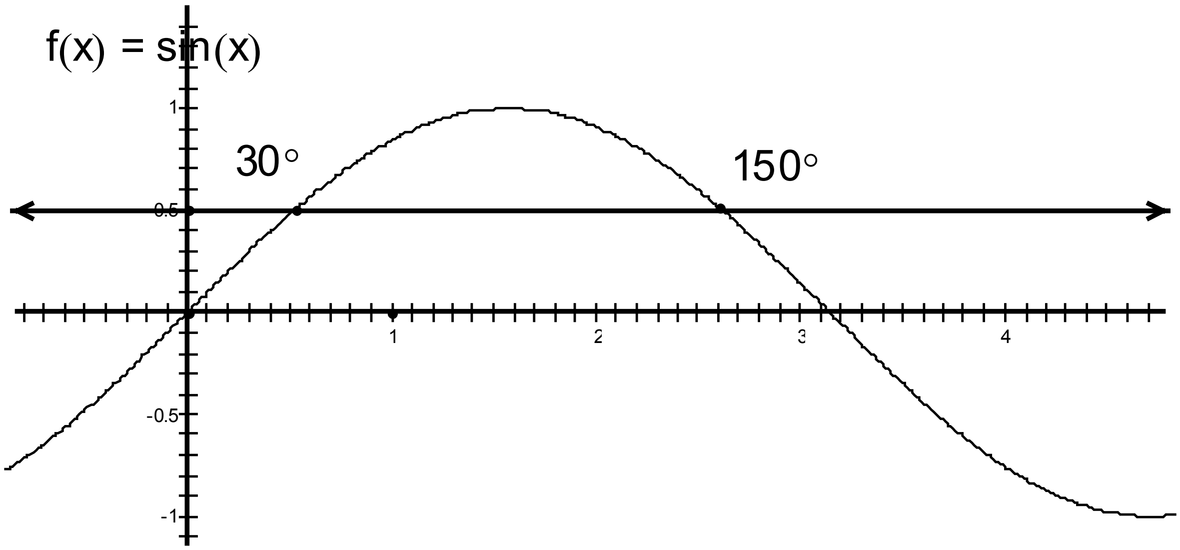
$$\sin^{-1}\left(\frac{1}{2}\right) = \left\{30^\circ + 360^\circ n, 150^\circ + 360^\circ n : n \in \mathbb{Z}\right\}$$

Two Graphical ways to interpret this last result:

Sine is the Y coordinate in the unit circle view so we see where a Horizontal line intersects the circle



Here the output from sine is on the Y axis so again a horizontal line intersects as the solution



Inexact, using a calculator

Example: What is the angle whose sine is .356?, $\sin^{-1}(.356) = \_\_\_\_\_\_\_\_\_$
Set your Calculator's mode to Degrees.

FOR INVERSE FUNCTIONS THE MODE DETERMINES THE OUTPUT~

$\sin^{-1}(.356) = 20.85°$,

As in the previous example 180°-20.85°=159.15°.

$\sin^{-1}(.356) = \{20.85^\circ + 360^\circ n, 159.15^\circ + 360^\circ n : n \in \mathbb{Z}\}$

TRY FIRST 5 Problems on hand out

Go Over these problems

Finding the Inverse for ctn(), sec() and csc() with a Calculator!

Note that your calculator does not have a button for Inverse Cotangent, Secant or Secant.

Example: Find $ctn^{-1}(-3.8) = ?$

$ctn^{-1}(x) = y$	(Take the Inverse of both sides
$x = ctn(y)$	(Take the reciprocal of both sides
$\frac{1}{x} = \frac{1}{ctn(y)} = \tan(y)$	(Find the inverse tangent of both sides
$\tan^{-1}\left(\frac{1}{x}\right) = y$	(Now substitute *y* from the first equation
$ctn^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$	(So we can find the inverse cotangent of *x* by finding the inverse tangent of $1/x$

Similarly

$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right) \qquad \text{and} \qquad \csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$$

TRY Problems 6 and 7 on the hand out

Go Over these problems

More complicated Problems:

Find all solutions to the following: $\sin(3\theta) = 0.469$ for $0^\circ \le \theta \le 360^\circ$.

First note that $\sin^{-1}(.469) = 27.97^\circ$ In Quadrant I

So $\theta = \dfrac{27.97^\circ}{3} = 9.32$

But the sine function is also positive in Quadrant II giving the solution:

$$\frac{180^\circ - 27.97^\circ}{3} = 50.68$$

Next we can try

$$\frac{360^\circ + 27.97^\circ}{3} = 129.32$$

And

$$\frac{360^\circ + 180^\circ - 27.97^\circ}{3} = 170.68$$

Continuing

$$\frac{2(360^\circ) + 27.97^\circ}{3} = 249.32$$

$$\frac{2(360^\circ) - 27.97^\circ}{3} = 290.68$$

$$\frac{3(360^\circ) + 27.97^\circ}{3} = 369.3 > 360$$ So we are done:

$\theta = \{9.32, 50.68, 129.32, 170.68, 249.32, 290.68\}$

You might notice that 9.32, 129.32, and 249.32 differ by 360/3 = 120 and 50.68, 170.68 and 290.68 also differ by 120.

Is this a coincidence?

Do problem 8 on the Handout