Lesson Plan 10 Solving Triangles Math 48C Mitchell Schoenbrun

1) Attendance
2) Collect Home Work

What do I mean by SOLVING TRIANGLES?

Geometry gives us four Triangle Congruence Theorems. Actually one of them is usually a postulate.

SSS
SAS
ASA
AAS
HL

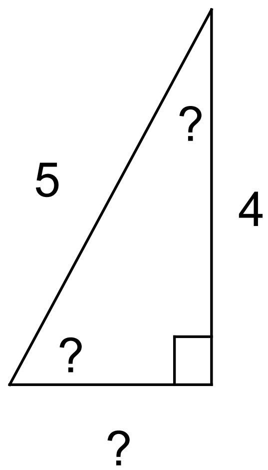
Note that AAS is equivalent to ASA because if two angles of a triangle are the same, the third must be the same.

These theorems give us a basis for determining if two triangles are congruent (Same length sides, same angles) based on their having three features in common.

But this also means that if you know these three features of a triangle, that triangle is completely determined.

What SOLVING A TRIANGLE means is once we have any three features, determine the others.

From Geometry we already have some tools that help us do this. We know this triangle is determined by HL Hypotenuse Leg

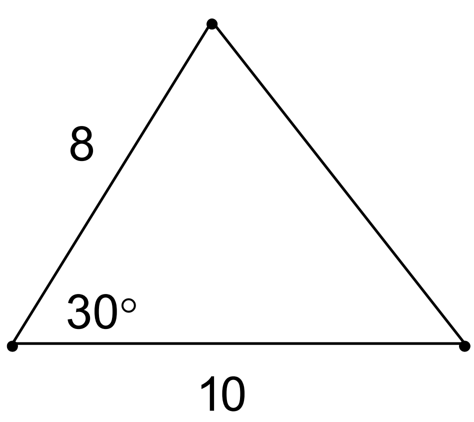


Using the Pythagorean Theorem, we can find the missing leg.
Then using our inverse trig functions, we can find one of the missing angles.

$$\cos^{-1}\left(\frac{4}{5}\right) = ?$$

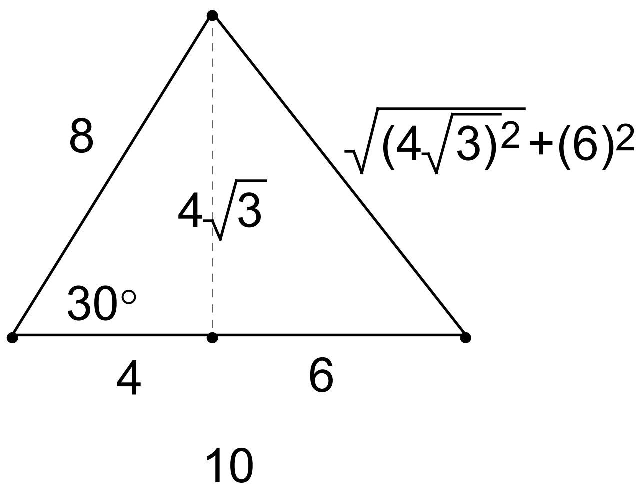
Finally we can find the last angle in multiple ways.
(Inverse Trig Functions)
(Sum of the Angles)
(Non right angles in a right triangle are complementary

For a more general triangle, we still can do the job, eg.



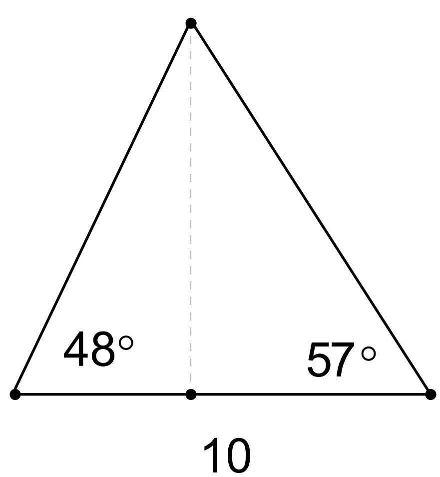
What can we do here? SAS

Drop an altitude!



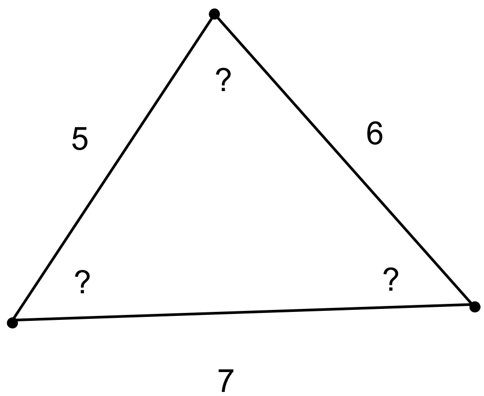
Now we have 2 right triangles and we can figure out the angles as before.

But what about this situation? ASA



Note that dropping an altitude doesn't help much.

Or for SSS



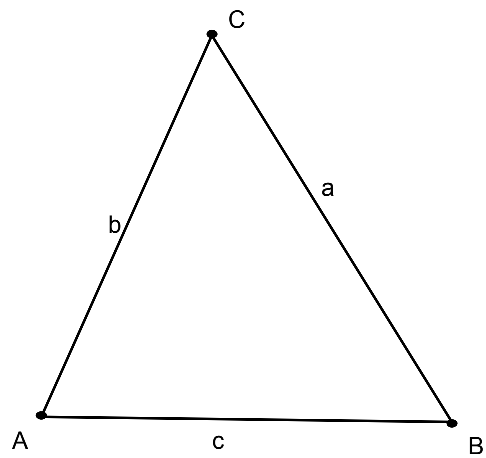
We need some better tools!

Two New Theorems You will Need to know.

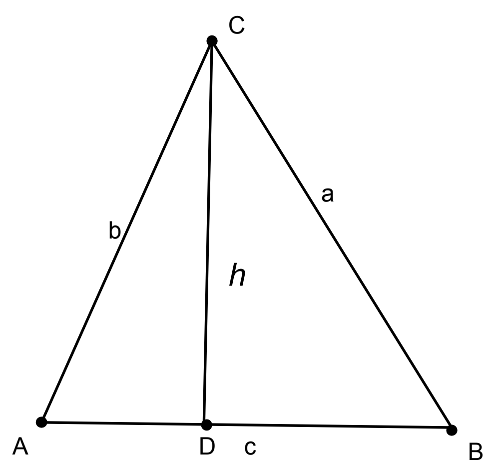
1) The Law Of Sines

2) The Law of Cosines

First we show derive the law of Sines!   Take a general acute triangle.



Drop an altitude of length h



Now Note the following:

$\frac{h}{b} = \sin A$ $\frac{h}{a} = \sin B$

multiplying by b and a respectively:

$h = b\sin A$ $h = a\sin B$

By transitivity we have

$b\sin A = a\sin B$ or after dividing both sides by ab you get

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

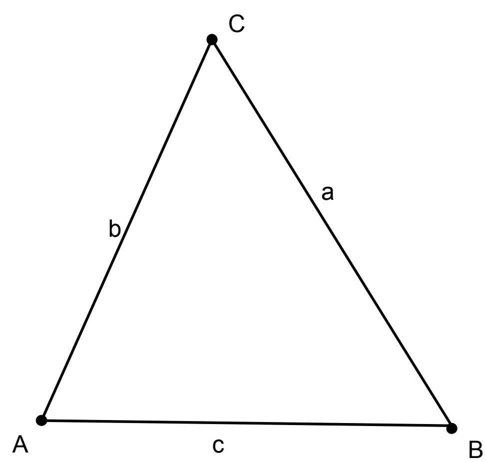
Repeat this all for an altitude dropped from A or B and you get the complete Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

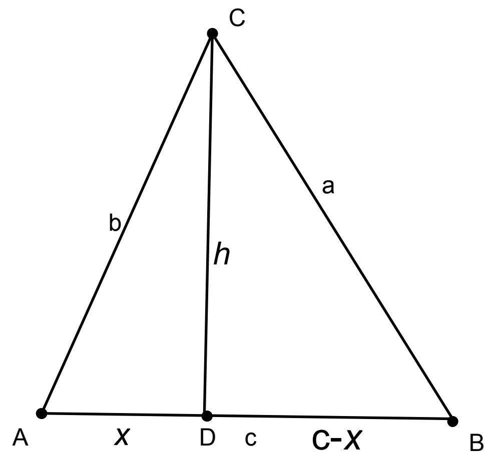
THIS IS THE LAW OF SINES

So given any two angles and a cooresponding side, or any two sides and a cooresponding angle, the missing angle or side can be calculated!

Now the Law of Cosines!



Drop an altitude length h, dividing c into x and (c-x)



Note the following trigonometric relationship:

$$\cos(a) = \frac{x}{B}$$

Multiplying by B we get

$B\cos(a) = x$ We will use this later

Now use the Pythagorean theorem on the two right triangles getting

$$B^2 = h^2 + x^2$$

and

$$A^2 = h^2 + (C - x)^2$$

solve both equations for h^2

$h^2 = B^2 - x^2$ and $h^2 = A^2 - (C - x)^2$

by transitivity

$$B^2 - x^2 = A^2 - (C - x)^2$$

Multiply $(C - x)^2$ and simplify

$$B^2 - x^2 = A^2 - (C^2 - 2Cx + x^2)$$

$$B^2 - x^2 = A^2 - C^2 + 2Cx - x^2$$

$B^2 = A^2 - C^2 + 2Cx$
And now Solve for A^2

$A^2 = B^2 + C^2 - 2Cx$

Now we substitute in x from above and get

$A^2 = B^2 + C^2 - 2BC\cos(a)$

Note that we could do this derivation using any permutation of the sides so we also have

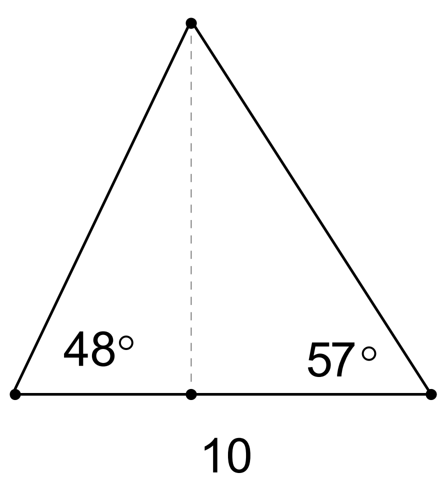
$B^2 = A^2 + C^2 - 2AC\cos(b)$

and

$C^2 = A^2 + B^2 - 2AB\cos(c)$

This is the LAW OF COSINES!

How can we use these? Let's go back to the previous two unsolved problems
ASA



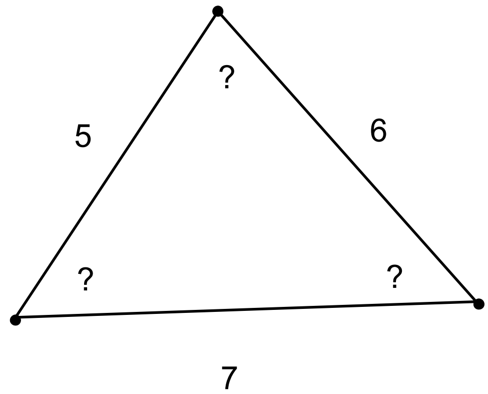
We can easily find that the third angle is 180° - 105° = 75°.

Using the Law of Sines we have

$$\frac{\sin(75^\circ)}{10} = \frac{\sin(57^\circ)}{b} = \frac{\sin(48^\circ)}{a}$$

We can solve for a or b by cross multiplying and dividing!
a = 7.69 b = 8.68

For this problem SSS we use the Law of Cosines



To find the angle across from the 7 side we note that:

$$7^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cos(x)$$

Solving for x we get

$$x = \cos^{-1}\left(\frac{7^2 - 6^2 - 5^2}{-(2 \cdot 5 \cdot 6)}\right) = 78.46^\circ =$$

In the same way we can find the other two angles

Give students handout with problems