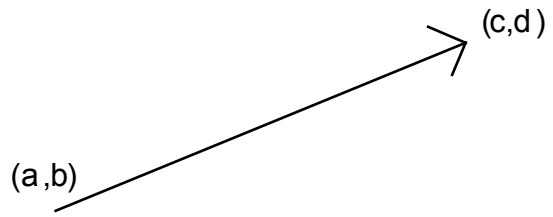
Lesson Plan 13 Vectors Math 48C Mitchell Schoenbrun

Note this approach is a little different from the book. I think it is more

1) Arrows

An arrow is a directed line segment. It has a starting point (or head) and an ending point (or tail). We draw an arrow as follows:



Note that by adding or subtracting the same value from the x and/or y coordinates, the new arrow has the same length and direction. The length is sometimes called the MAGNITUDE of the arrow.

(Show This)

There are an infinite number of different arrows that have the same length and direction.

These arrows are each EQUIVALENT, so the set of all such arrows is called an equivalence class.

2) VECTORS

A vector is an equivalence class of arrows. Note that like an arrow, it has a magnitude and a direction.

Any arrow that is a member of the equivalence class can be used to represent the vector. So we can choose an arrow to represent a vector that best meets our needs.

In the future I will draw an arrow but refer to it as a vector. Keep in mind that the arrow is a member of the equivalence class of arrows and that any of these arrows can be used to represent the vector.

A symbol for a vector is a letter with an arrow over it, eg. $\vec{V}$.

To symbolize a vector with the same magnitude as $\vec{V}$ but in the opposite direction, we will write it as $-\vec{V}$.

Later we will see that this is the same as multiplying the vector by -1, eg. $(-1)\vec{V}$

Note that two vectors are parallel if (and only if) they have the same or opposite direction.

Show this.

3) A vector at the origin.

One can always choose an arrow to represent a vector whose starting point is at the origin. Consider the vector

(a,b) ----> (c,d)

This can be represented as (0,0) ---> (c-a, d-b)

We can write this as just $\langle c-a, d-b \rangle$.

The book also calls this COMPONENT FORM.

We will use this form from now on.

4) The Magnitude and direction of a vector.

Using the Pythagorean theorem, we find that the Magnitude of vector $\langle a,b \rangle$

is $\sqrt{a^2+b^2}$ (Show this on the board

Using our knowledge of trigonometry, we find that the direction of a vector is

$\tan^{-1}\left(\frac{b}{a}\right)$ if the head of the vector is in quadrant I or III. Otherwise

$\tan^{-1}\left(\frac{b}{a}\right)+180^\circ$

5) Unit Vectors

A Unit Vector is a vector with magnitude 1.

6) Zero Vectors

The zero or NULL Vector is a vector with magnitude 0. We will write it as $\vec{0}$.
The zero vector is considered to have every direction.

7) Multiplying a vector times a constant or a SCALAR.

When talking about vectors, we refer to simple numbers as scalars.

We can multiply a vector times a scalar.

A) If the scalar is positive, we multiply the magnitude of the vector by this number and keep the direction the same to get the new vector. This can increase or decrease the magnitude of the vector.

B) If the scalar is negative, we multiply the magnitude of the vector by the absolute value of the number and reverse the direction.

C) If the scalar is zero, the new vector is the zero vector.

If we multiply a vector times a constant K then

$K\langle a,b\rangle = \langle Ka, Kb\rangle$

8) Adding vectors.

We add vectors geometrically by placing the head of one vector at the tail of the other. The new vector has as it's tail the tail of the first and it's head as the head of the second.

Show geometrically why this is commutative.

Show that the sum of $\langle a,b\rangle$ and $\langle c,d\rangle$ is $\langle a+c, b+d\rangle$.

9) Subtracting vectors.
Subtract one vector from another by adding it's opposite.

$$\vec{V} - \vec{U} = \vec{V} + \left(-\vec{U}\right)$$

Draw the vector parallelogram and show that both the sum and difference of two vectors is the diagonal of the parallelogram.

Problems)

1) Displacement Vectors

A traveler knows that he needs to to 5 miles north and 10 miles east to get to his destination.

If he were to fly directly there, how far and in what direction should he go?

2) Velocity.

A jet is flying from North/East at a 30 degree angle at 600 miles per hour with no wind.
The jet encounters a wind blowing south at 100 miles per hour.
What direction should the jet head in order to travel in the same direction?
What will the ground speed be?

3) Force

A motionless object always has a net force on it of zero.
A ball that weighs 10 lbs is held up by a wire.
Another wire pulls the ball horizontally so that the angle with the vertical is 30 degrees.
What are the forces on the two wires?

Hand out some problems to work on in class.