

Lesson Plan 14 Vectors-Redo Math 48C Mitchell Schoenbrun

Vectors:

A vector is a mathematical object with both a magnitude and a direction.

We represent a vector graphically using an arrow with a starting point called the tail and an ending point called the head.

This can be described by two ordered pairs $(a,b) \rightarrow (c,d)$

A displacement vector is one whose tail is at the origin.

You can find a displacement vector from a pair of head and tail points as follows:

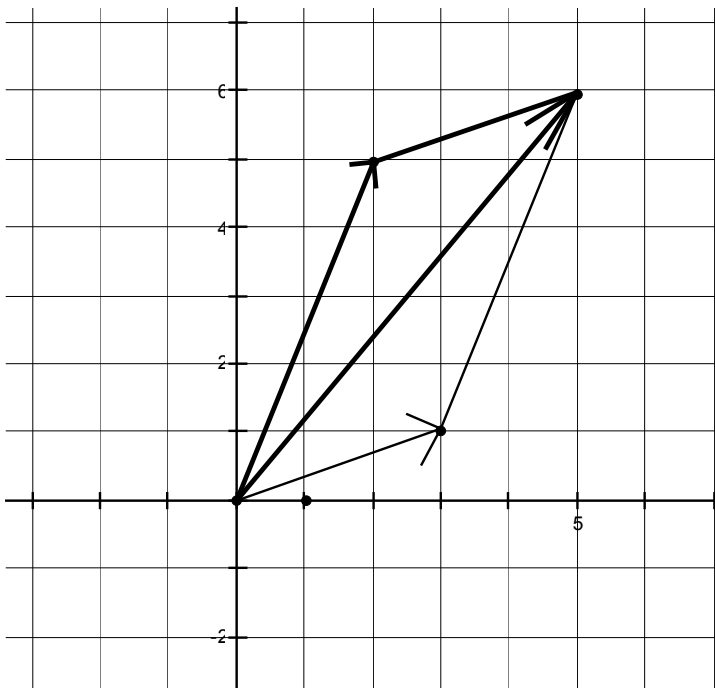
$(0,0) \rightarrow (x,y)$

$$x = c - a$$

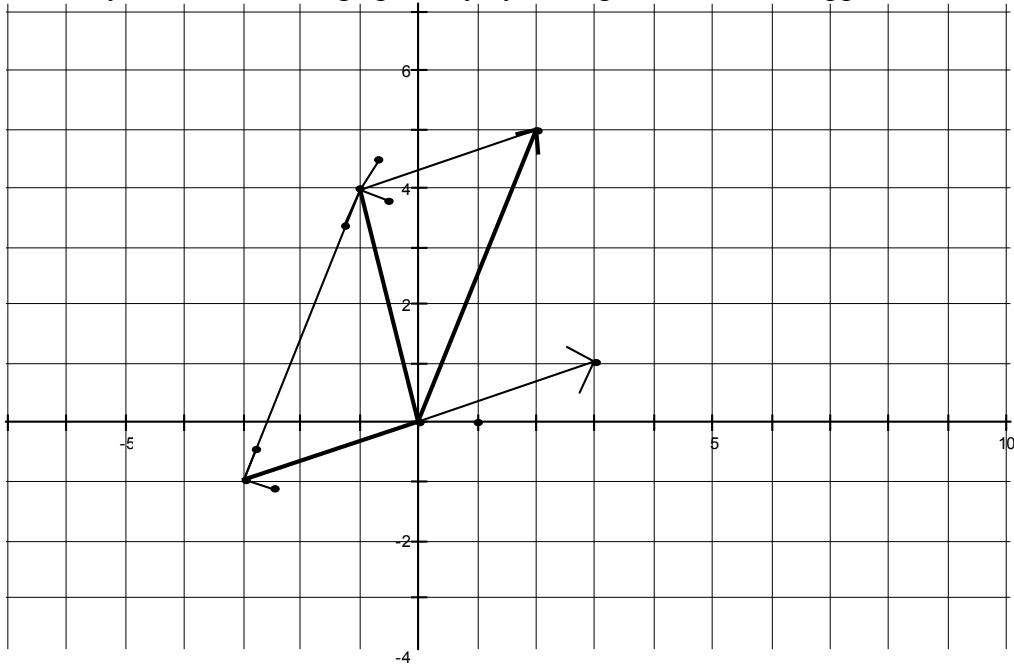
$$y = d - b$$

We write it this way with angle brackets $\langle x, y \rangle$ and call it a displacement vector in component form.

We add vectors graphically by moving the tail of one vector to the head of the other.



Similarly we can subtract graphically by adding a vector in the opposite direction.



When vectors are in component form, they are easy to manipulate:

Addition:

$$\vec{V} + \vec{U} = \langle V_x, V_y \rangle + \langle U_x, U_y \rangle = \langle V_x + U_x, V_y + U_y \rangle$$

Subtraction

$$\vec{V} - \vec{U} = \langle V_x, V_y \rangle - \langle U_x, U_y \rangle = \langle V_x - U_x, V_y - U_y \rangle$$

Multiplying by a constant

$$k\vec{V} = k \langle V_x, V_y \rangle = \langle kV_x, kV_y \rangle$$

Finding a vector in the opposite direction

$$-\vec{V} = (-1) \langle V_x, V_y \rangle = \langle -V_x, -V_y \rangle$$

Note the Zero vector has any direction

$$\vec{O} = \langle 0, 0 \rangle$$

Just like adding zero to a number, you have

$$\vec{V} + \vec{O} = \langle V_x, V_y \rangle + \langle 0, 0 \rangle = \langle V_x + 0, V_y + 0 \rangle = \langle V_x, V_y \rangle = \vec{V}$$

Pass Out Handout, Do problems 1-4

Problems Using Vectors

Compute the Magnitude and direction of Vector $\langle 2, 5 \rangle$

A plane traveling 200 miles south at 80 miles per hour.
Indicate the vector in component form and calculate the flight time.

An airplane travels 80 per hour south.
It encounters a 25 mph wind blowing east.

How from the destination will he end up after 2.5 hours.

Determine the heading so he will reach the destination.

How long will it take?

Multiply the vector $\langle 3, 6 \rangle$ by $1 + 1/18$

Subtract two vectors $\langle 10, 12 \rangle - \langle 4, 7 \rangle$

Definition A unit vector.

A unit vector is a vector with magnitude 1.

We define two vectors as follows:

$$\vec{i} = \langle 1, 0 \rangle$$

$$\vec{j} = \langle 0, 1 \rangle$$

These can be used to decompose a vector as follows:

$$\langle 5, 4 \rangle = 5\langle 1, 0 \rangle + 4\langle 0, 1 \rangle = 5\vec{i} + 4\vec{j}$$

Because any vector in the plane can be decomposed this way, these two vectors are called a basis for the space.

In addition, because the two vectors are perpendicular (at 90° angle) they are called **orthogonal**.

Last Problem:

When dealing with forces, if an object is motionless, then the total vector force on it is zero.

Example: A 400 pound force is required to hold down a hot air balloon. Two ropes at 45 and 25 degrees are used to tether the balloon in place. What is the force needed on each balloon.

Have students try Problems 5 and 6 on handout.