Lesson Plan 15 Vectors-Advanced Math 48C Mitchell Schoenbrun

1) Attendance
2) Go over any homework questions
3) Go over some of the odd problems on the homework

So far Vectors have similarities to real numbers:

Properties
1) Closure
2) Associativity
3) Additive Identity
4) Additive Inverse
5) Commutativity

Vectors are also closed under Scalar multiplication
Vectors are also associative under Scalar multiplication
The value 1 is a scalar multiplicative identity
Zero times any vector gives the zero vector
Vectors are distributive in two ways

$$k\left(\vec{V}+\vec{U}\right)=k\vec{V}+k\vec{U}$$

$$\left(k_1+k_2\right)\vec{V}=k_1\vec{V}+k_2\vec{V}$$

A set of vectors that obey these rules are called a Vector Space.

New Vocabulary plus Notation:

The NORM of a vector $\vec{V}$ is indicated $\left\|\vec{V}\right\|$

The NORM of a vector is the same as it's magnitude.

So if $\vec{V}=\langle 3,4\rangle$ then $\left\|\vec{V}\right\|=\sqrt{3^2+4^2}=5$

Note the triangle inequality holds for any two vectors: $\left\|\vec{V}+\vec{U}\right\|\le\left\|\vec{V}\right\|+\left\|\vec{U}\right\|$

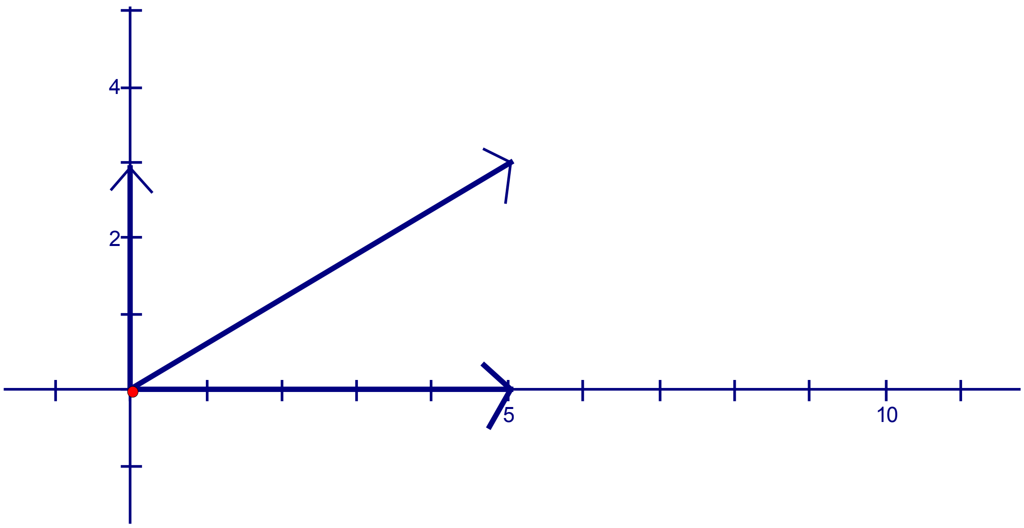
Explore this graphically

As mentioned before, two vectors are ORTHOGONAL if they are at right angles.
A curious fact is that

Two vectors are orthogonal if and only if $\left\|\vec{V}+\vec{U}\right\| = \left\|\vec{V}-\vec{U}\right\|$

Draw a rectangle and show why this is so!

In some problems it is helpful to resolve a vector into it's components. This can be done by PROJECTING the vector onto an another vector.



The magnitude of the projection is $\left\|\vec{V}\right\|\cos(\theta)$ where θ is the angle between the vectors.

So for a vector $\vec{V}$ we have $\vec{V} = V_x i + V_y j$ where

$$V_x = \left\|\vec{V}\right\|\cos(\theta)$$

$$V_y = \left\|\vec{V}\right\|\cos(90-\theta) = \left\|\vec{V}\right\|\sin(\theta)$$

Now we introduce a way of multiplying two vectors that produces a scalar, called the DOT PRODUCT

$$\vec{V}\cdot\vec{U}$$

We have two ways to calculate the dot product

$\vec{V}\cdot\vec{U} = \left\|\vec{V}\right\|\left\|\vec{U}\right\|\cos(\theta)$ where θ is the angle between the vectors, or

$$\vec{V}\cdot\vec{U} = V_x U_x + V_y U_y$$

Since these are equivalent we have

$\|\vec{V}\|\|\vec{U}\|\cos(\theta) = V_x U_x + V_y U_y$

This can be really useful

Problem 1

What are the vertical and horizontal projections of the vector $\langle 3,5 \rangle$ onto the axes

Problem 2

What is the angle between the vectors $\langle 7,6 \rangle \, and \, \langle -2,5 \rangle$

Problem 3

Are the vectors above orthogonal

Use both methods to figure out $\|\vec{V}+\vec{U}\| = \|\vec{V}-\vec{U}\|$ and
$\|\vec{V}\|\|\vec{U}\|\cos(\theta) = V_x U_x + V_y U_y$

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What is a Basis?

A basis is a set of vectors $\{\overrightarrow{V_1}, \overrightarrow{V_2}, \ldots, \overrightarrow{V_n}\}$
such that for every vector you have

$\vec{V} = A_1\overrightarrow{V_1} + A_2\overrightarrow{V_2} + \ldots + A_n\overrightarrow{V_n}$
and
if $A_1\overrightarrow{V_1} + A_2\overrightarrow{V_2} + \ldots + A_n\overrightarrow{V_n} = 0$ then $A_1 = A_2 = \ldots = A_n = 0$

Show that $\{i, j\}$ is a basis for E^2

Note that $\{i, j\}$ is also an orthogonal basis since

$$i \cdot j = \langle 1,0 \rangle \cdot \langle 0,1 \rangle = 1 \cdot 0 + 0 \cdot 1 = 0$$