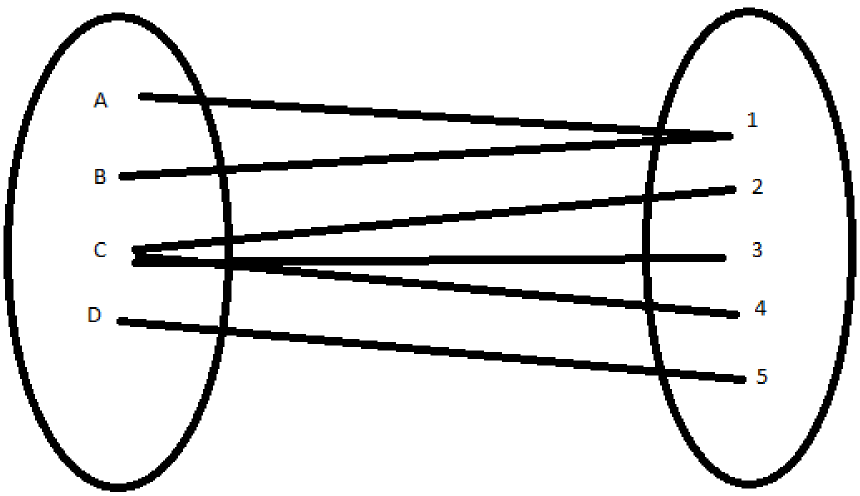
Lesson Plan 8 Inverse Trig Functions Math 48C Mitchell Schoenbrun

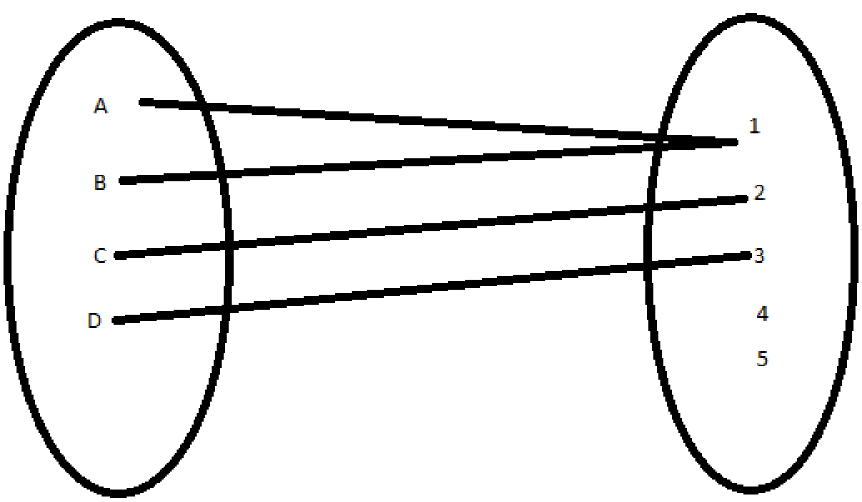
1) Attendance
2) Hand out homework.
3) Questions about homework?
4) Quiz on 8.4 and 8.5 Graphing functions/Modeling Functions, Angular Motion makeup

Define a Relation and a Function

A relation is a mapping beween two sets

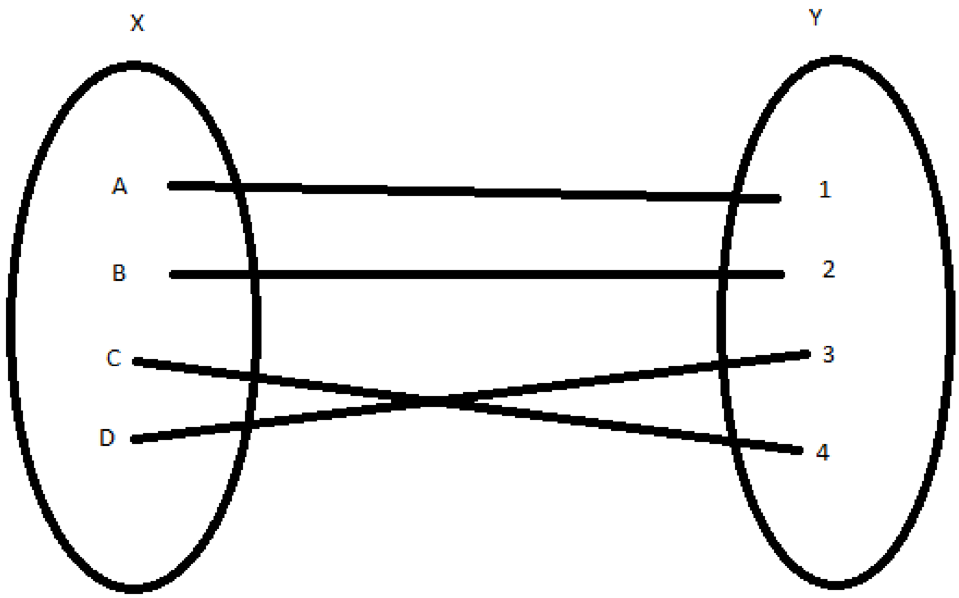


Note that this relationship is NOT a function.   Why?

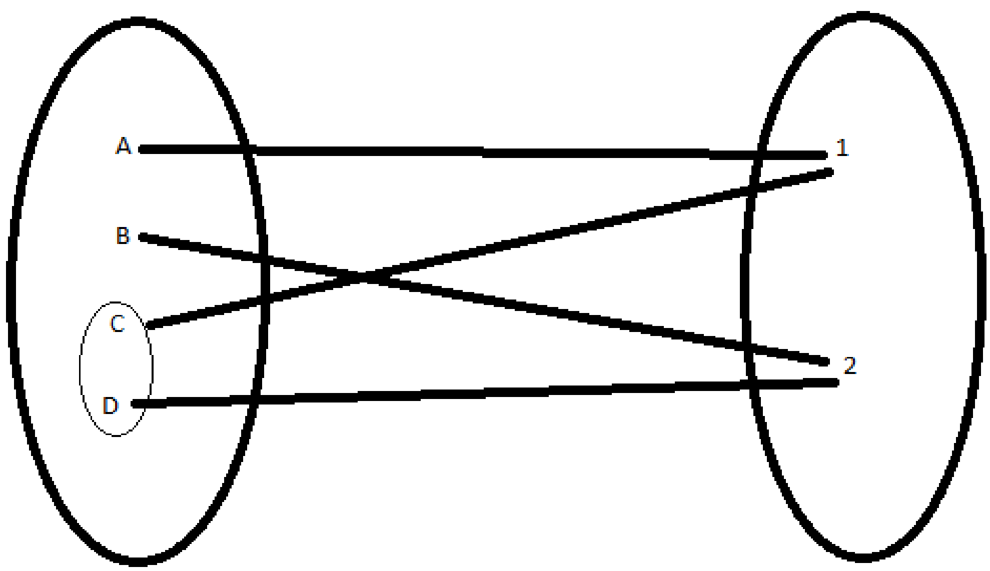


This relation is a function.   Why?

This is a function from X ---> Y with an inverse from Y----> X

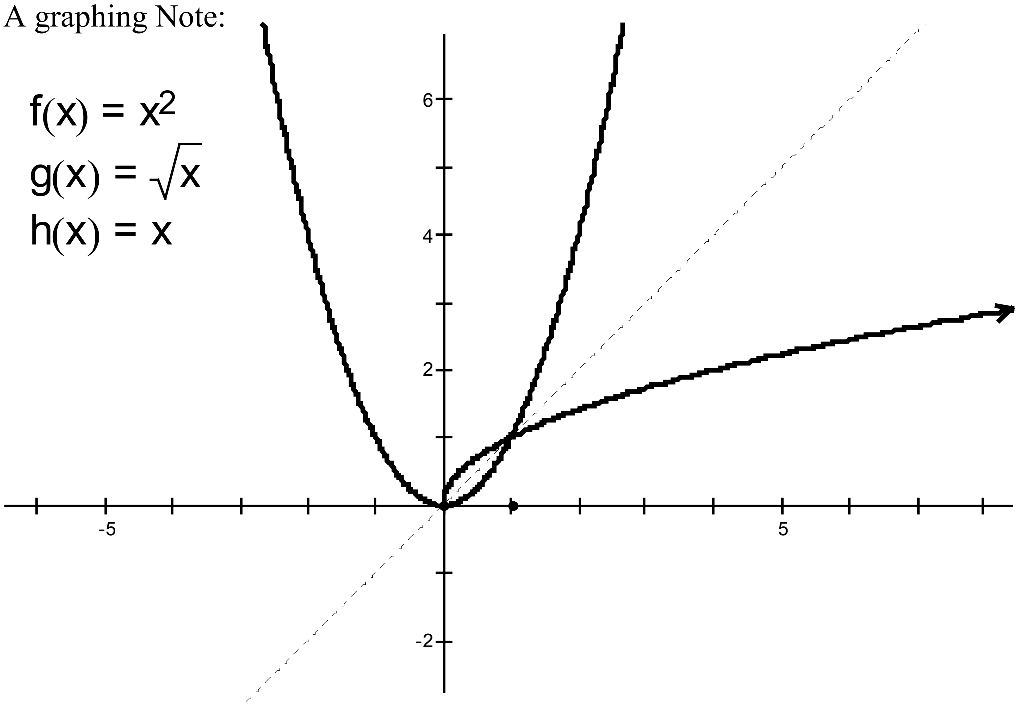


Some sometimes can give a function an inverse by truncating the Domain

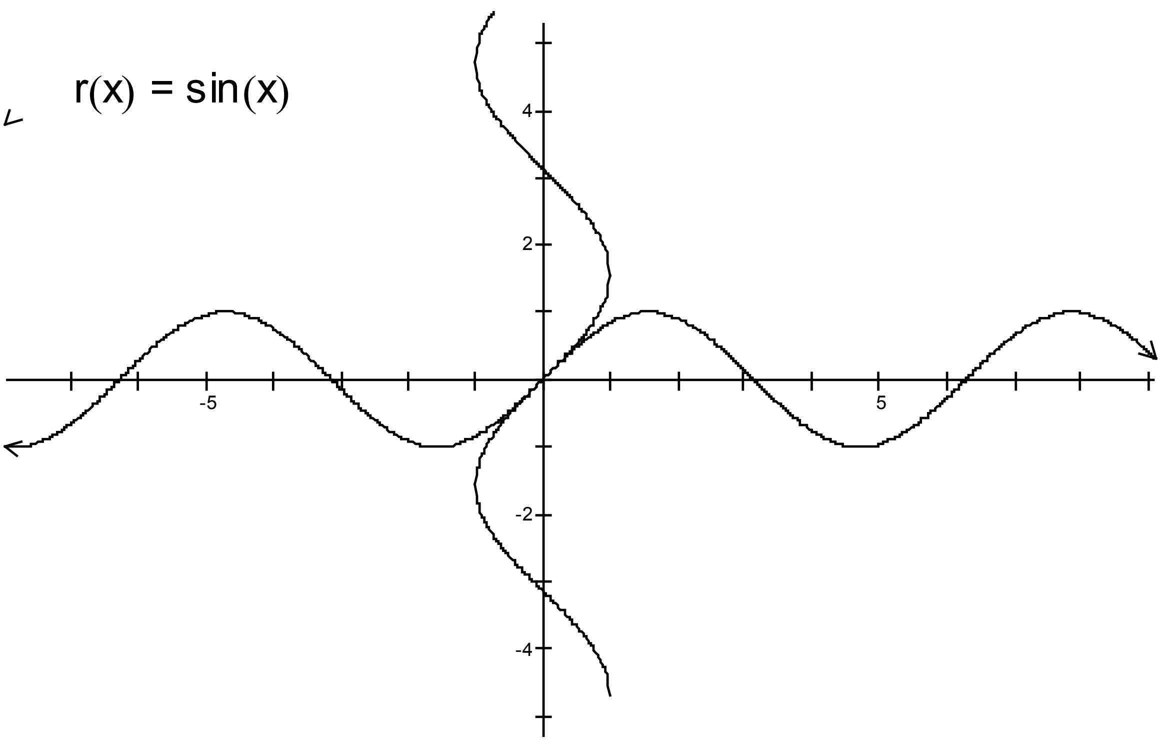


Note that by removing C and D from the domain, we now have a function that has an inverse.

A graphing Note:



You can graph the inverse function by rotating along the line y=x.



If you do this with the sine function, the result is not a function anymore.

So we must truncate the Domain of Sine so that it's inverse exists.

We truncate the Domain to be $[-\pi, \pi]$

A word on Notation. [A, B] is a closed interval from A to B. It includes the end points A and B

(A,B) is an open interval from A to B. It includes all the points between A and B but not A or B

You can have half open, half closed intervals eg. [A, B)

To describe all positive real numbers you would use this notation:

$[0, +\infty)$

Note that intervals with infinite limits are always considered open.

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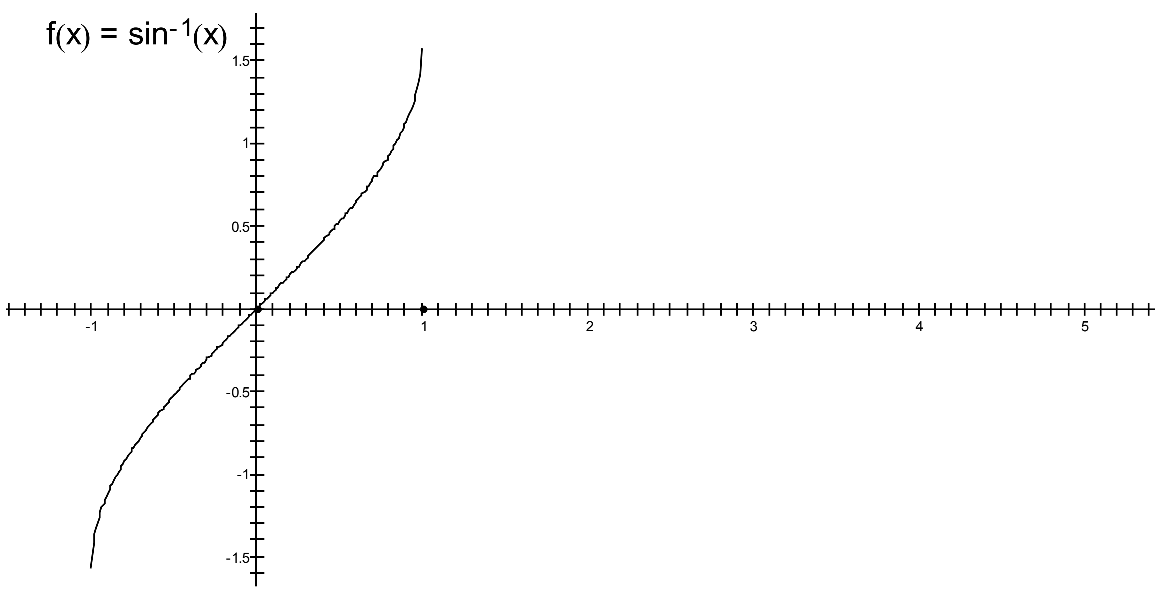
Back to The Sine Function and it's inverse. If we limit the Sine function to $[-\pi, \pi]$. The inverse of the Sine function is written either

$$\arcsin(x)$$
$$or$$
$$\sin^{-1}(x)$$



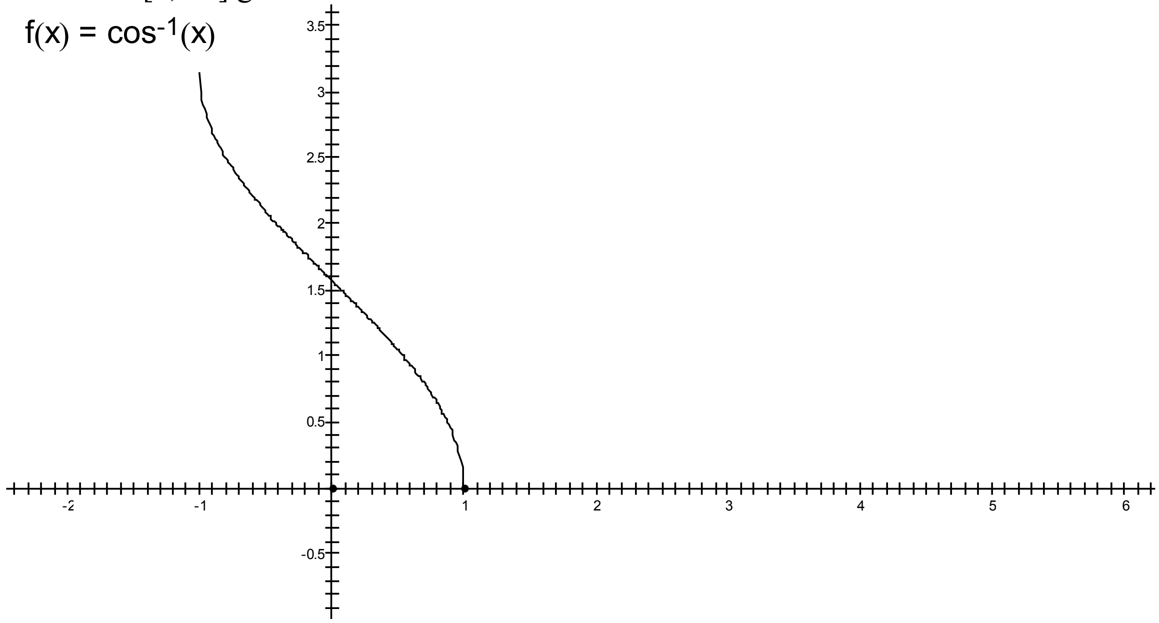
The domain of this function [-1, 1] is the range of it's inverse.
The range of this function $[-\pi, \pi]$ is the domain of it's inverse.

You can use your calculator to find specific values of an inverse function:

Note that the resulting value will depend on the MODE of the calculator
Degrees or Radians.

The Cosine's domain must be truncated a little differently to get an inverse.

Domain = [0, 2π] gives us the inverse cosine as follows:

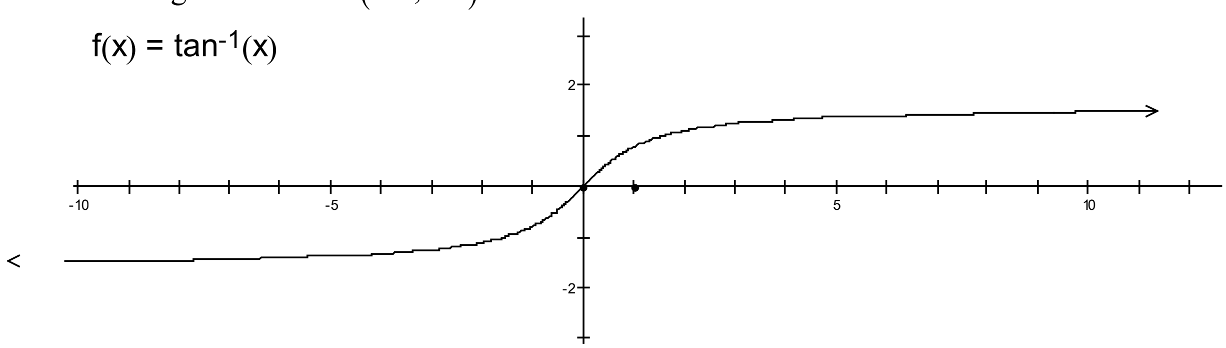


Note that it's Domain is [-1,1] like the arcsine, but it's range is [0,π]

With the Tangent function, we now restrict the Domain to be $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Note that this is an open interval. The end points are undefined.

The range however is $(-\infty, +\infty)$



The functions described above can help you find the angle whose sine, cosine, or tangent is a specific value.

However you should be aware that this angle is not unique.

Example:

Find the angle whose sine is $\frac{1}{\sqrt{2}}$

Since $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

However the sine is positive in both the first and 2nd quadrants, so

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

But since the sine is periodic we have multiple solutions:

$$\left\{\frac{\pi}{4} + 2\pi n, \frac{3\pi}{4} + 2\pi n\right\} \textit{where}\, n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

Look at Handout