Math 109 Calc 1 Lecture 0

Review

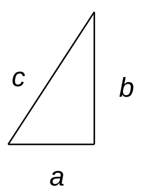
Hopefully for most of you this will be a boring tedious lecture where I go over things you already know well. If not, please pay attention to anything you don't recall. Please stop me if there's something you need explained. If it's been a while since you've taken a math class, you may want to ask me for my notes to review them yourselves. Assume that if you don't know this information, you need to do some extra work. Otherwise you will find this class especially difficult.

Geometry

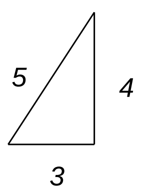
The most important result from Geometry that you need to be aware of is the **Pythagorean Theorem**

$$a^2 + b^2 = c^2$$

Where c is the length of the hypotenuse of a right triangle and a and b are lengths of the legs.



There are a few specific triangles that you should be familiar with.

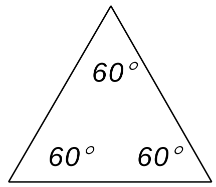


One useful triangle is a **3,4,5 right triangle**. It's easy to see that

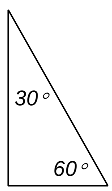
$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

This type of triangle shows up in problem sets a lot.
In addition, any multiple of these such as 6, 8, 10 or 9,12, 15 will be a right triangle.

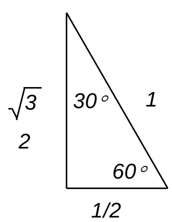
Another triangle you should be familiar with is an **equilateral triangle** whose sides are all equal in length and whose angles are all 60 degrees, or



By dropping a perpendicular from the apex of this triangle, one gets another important triangle the **30/60/90 triangle**.



It's important to recognize this triangle and to know that the sides are in a ratio of
$1/\frac{1}{2}/\frac{\sqrt{3}}{2}$



This triangle also comes up quite often in problems.

You should know how to calculate the area and volume of some basic shapes.

Area of a rectangle
Area of a triangle
Area of circle

Volume of a parallelepiped
Volume of a cylinder
Volume of a cone
Volume of a sphere

Algebra

You should know how to solve a simple **linear equation** in 1 unknown.

Example: $5x + 7 = 3$

You should know how to solve **two linear equations in two unknowns**.

Example: $3x + y = 7$ $\quad 2x - y = 4$

You should know what a **polynomial** $P(x)$ and a **polynomial equation**, $P(x) = 0$ are, what the degree of a polynomial is, what a term, coefficient, leading coefficient, and constant term are.

Example:
A polynomial: $P(x) = x^3 + 2x^2 - x + 5$

A polynomial equation: $x^3 + 2x^2 - x + 5 = 0$

The degree is 3

The terms are x^3, $2x^2$, $-x$ and 5

2 is the coefficient of $2x^2$

The leading coefficient is 1

The constant term is 5

You should know what the **root** or **zero** of a polynomial equation is, a value that is a solution to the equation.

Example: $x^2 - 2x + 1 = 0$ has a root of $x = 1$

You should know the **quadradic formula** and that it can always find the roots of a **quadratic equation**:

For the equation $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: $x^2 - 3x + 2 = 0 \quad x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)} = \dfrac{3 \pm \sqrt{9-8}}{2} = \dfrac{3 \pm 1}{2} = 1, 2$

You should know what an **imaginary number** is and what a **complex number** is.
You should also know what the **complex conjugate** of a complex number is.

Example: $\sqrt{-1} = i$ is an imaginary number and 3+2i is a complex number. Its complex conjugate is 3-2i.

You should know that the **discriminant** $b^2 - 4ac$ can tell you how many real and/or complex roots the equation has.

Example: $x^2 + x + 1 = 0 \quad b^2 - 4ac = 1^2 - 4(1)(1) = -3$ which is negative, so the equation has two complex roots.

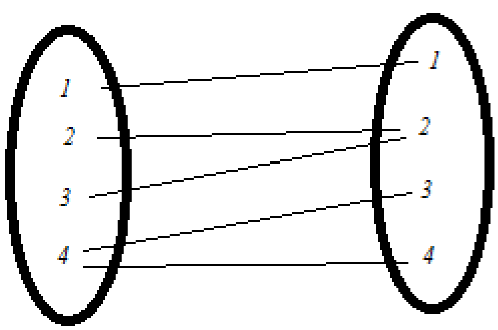
You should know how to divide polynomials using long division and optionally **synthetic division** when appropriate.

$$\frac{x^3 + 6x^2 + 5x + 2}{x + 2}$$

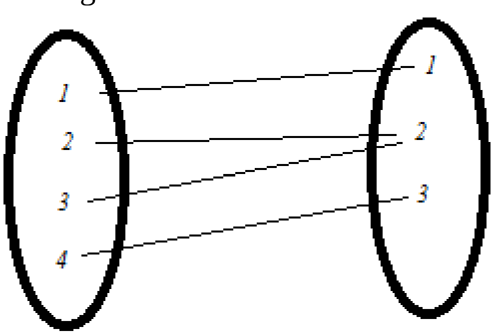
$$\begin{array}{r} x^2 + 2x + 1 \\ x + 2 \overline{\big) x^3 + 4x^2 + 5x + 2} \\ x^3 + 2x^2 \quad\quad\quad \\ 2x^2 \quad\quad\quad \end{array}$$

You should know what a **relation** a **function** and **one-to-one function** is.

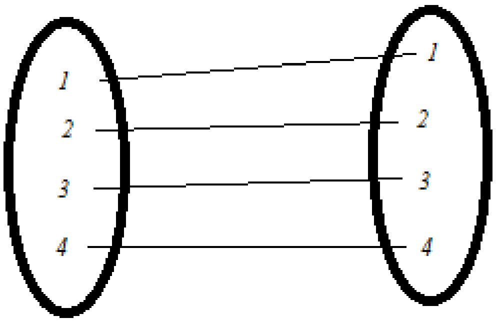
A relation is a mapping between two sets called the **domain** and the **range**.



A function is a relation where each element of the domain is mapped to only one element of the range.



A one-to-one function is one where each element of the domain maps to exactly one element of the range and each element of the range is mapped to by exactly one element of the domain.

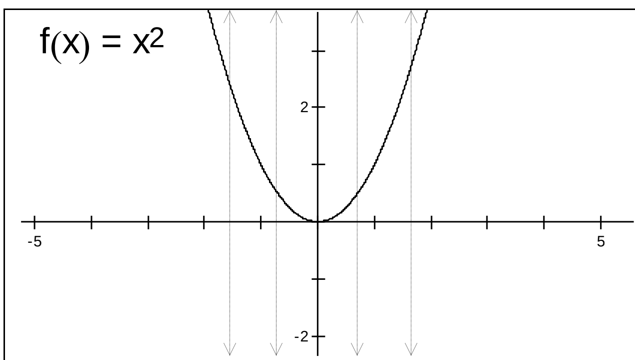


A function's mapping can be described by an equation.

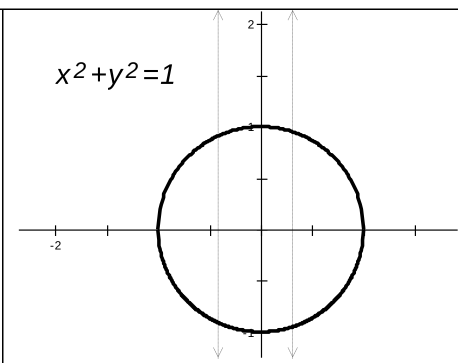
Example: $f(x) = x + 5$

If the expression is a polynomial, then the function is a polynomial function.

If you graph a relation, you can tell if it is a function using the **vertical line test**.

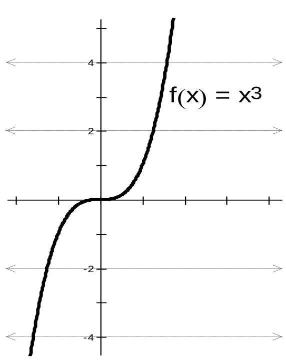


A function



A relation but not a function

The graph of a one-to-one function will pass the **horizontal line test**.



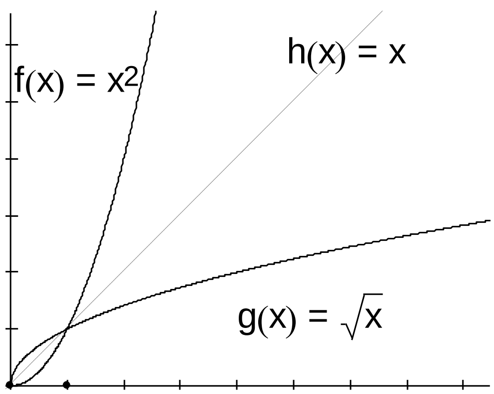
You should know what the **inverse function** of a function is.

A function $f^{-1}(x)$ is an inverse function if (and only if)

$$f^{-1}(f(x)) = f(f^{-1}(x)) = x$$

To have an inverse, a function must be one-to-one.

The graphs of a function and an inverse function will be mirror images across the line $y = x$



A note on domains

If the domain of a function is not specified, by convention we assume it is all real numbers where the function is defined.

Example: If not specified the domain of $\sqrt{x}$ is $x \geq 0$

Pre-Calculus

You should know the rational root theorem and the remainder theorem and how to use them when factoring a polynomial.

Example:

The only possible rational roots of $x^3 + 6x^2 + 5x + 2 = 0$ are $\left\{ \frac{2}{1}, \frac{-2}{1}, \frac{1}{1}, \frac{-1}{1} \right\}$

If you divide a polynomial function $f(x)$ by x - c, then the remainder is $f(c)$.

You should know how to factor a polynomial using reverse FOIL, the quadratic formula and the rational root theorem, and optionally be grouping.

You should know what an **increasing function** or **decreasing function** is.

An increasing function is one where if $a < b$ then for all a and b in the domain of the function.

An increasing function is one where if $a < b$ then $f(a) \geq f(b)$ for all a and b in the domain of the function.

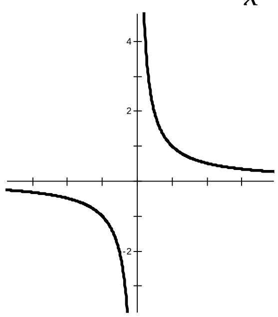
You should know what a **rational function** is.

A rational function is one in the form $f(x) = \frac{P(x)}{Q(x)}$ where P is a polynomial function and Q is a polynomial function of degree 1 or greater.

The domain of a rational function if not specified is assumed to be all real numbers where $Q(x) \neq 0$.

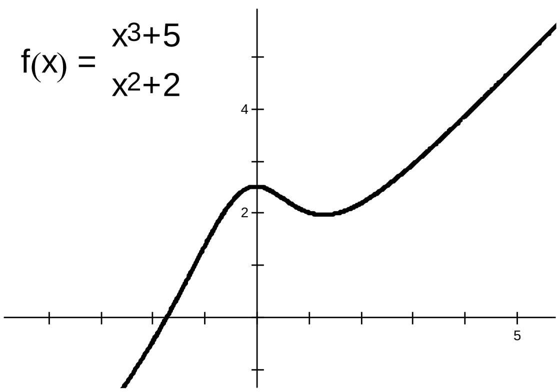
You should know how to find the **vertical asymptotes** of a rational function, the **horizontal asymptotes** if there are any, and the end behavior.

Example: $f(x) = \frac{1}{x}$ has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$.



Example: $f(x) = \dfrac{x^3+5}{x^2+2}$

This function has no asymptotes. Its end behavior is to increase as *x* gets large and to decrease as *x* gets large in the negative direction.



You should know what an **exponential function** is
Example: $f(x) = 10^x$

You should about log functions, the inverse functions of exponential functions.
Example: $f(x) = \log_{10} x$

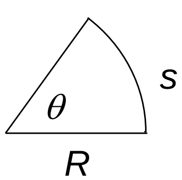
You should know the laws of exponents and the laws of logs.

You should know about the important mathematical constant ***e***

You should know that a **natural log** is a $\ln(x) = \log_e x$

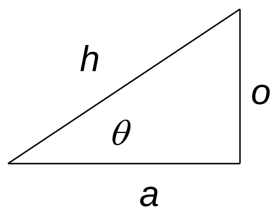
Trigonometry

You should know how **radian measure** is defined and how to convert from and to degrees.



$$\theta = \frac{s}{R}$$

You should know the triangle definitions of the 6 trigonometric functions



$$\sin(x) = \frac{o}{h}$$

$$\cos(x) = \frac{a}{h}$$

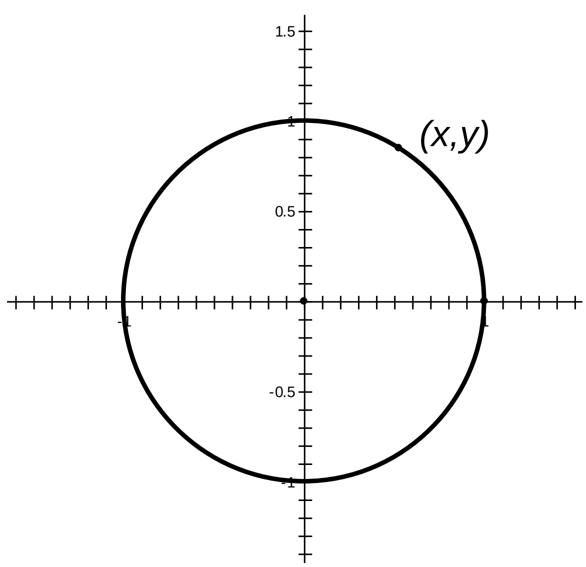
$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{o}{a}$$

$$ctn(x) = \cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{a}{o}$$

$$\sec(x) = \frac{1}{\cos(x)} = \frac{h}{a}$$

$$\csc(x) = \frac{1}{\sin(x)} = \frac{h}{o}$$

You should also know the circle definitions of the 6 trigonometric functions.



$$\sin(\theta) = y$$

$$\cos(\theta) = x$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{y}{x}$$

$$ctn(\theta) = \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{x}{y}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{x}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{y}$$

You should know the **period** of these functions.

Period 2π : sin(), cos(), sec(), csc()

Period π : tan(), ctn()

You should know what the graphs of these functions look like.

You should be able to find the values of some specific values without a calculator.

$$\left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi ...\right\}$$

Trig Identities and Formulas

There are a few important **trig identities** that you should know

The first of these is the Pythagorean identity

$$\sin^2(x)+\cos^2(x)=1$$
$$\sin(x\pm y)=\sin(x)\cos(y)\pm\cos(x)\sin(y)$$
$$\cos(x\pm y)=\sin(x)\sin(y)\mp\cos(x)\cos(y)$$

From these you can deduce the double and half angle formulas

$$\sin(2x)=2\sin(x)\cos(x)$$
$$\cos(2x)=\sin^2(x)-\cos^2(x)=2\sin^2(x)-1=1-2\cos^2 x$$

$$\sin\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos(x)}{2}}$$

$$\cos\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1+\cos(x)}{2}}$$

Conic Sections

It may be useful to know the equations of the following conic sections

A circle centered at (a,b) with radius r

$$(x-a)^2+(y-b)^2=r^2$$

A parabola with vertex at (a,b)

$(y-b)=k(x-a)^2$ - Vertical

$(x-b)=k(y-a)^2$ - Horizontal

An ellipse with center at (a,b) and A and B the semi-major and semi-minor axes

$$\frac{(x-a)^2}{A^2}+\frac{(y-b)^2}{B^2}=1$$

A hyperbola with center at (a,b)

$$\frac{(x-a)^2}{A^2}-\frac{(y-b)^2}{B^2}=1$$